ANALAYSIS OF HEAT TRANSFER ON NANOFLUID FLOW OVER A LINEARLY POROUS STRETCHING SHEET VIA RUNGE KUTTA METHOD

MSC RESEARCH THESIS

BY: CHALEW ESHETE WARE

FITCHE, ETHIOPIA
JUNE, 2022

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Title: Analysis of Heat Transfer on Nanofluid Flow over a Linearly Porous Stretching Sheet Via Runge Kutta Method

We, the advisors of this thesis, hereby certify that we have read the revised version of the thesis entitled "analysis of heat transfer on nanofluid flow over a linearly porous stretching sheet via Runge Kutta Fehlberg method with shooting technique"prepared under our guidance by Chalew Eshete Ware submitted in partial fulfilment of the requirements for the degree of Masters of in Mathematic (numerical analysis) Therefore, we recommend that the submission of revised version of the thesis to the department following the applicable procedures.

Advisor………………………………… Signature……………… Date……………….

Co-advisor………………………………Signature………………… Date………........

We, the undersigned, members of the Board of Examiners of the thesis by Chalew Eshete Ware, have read and evaluated the thesis entitled "analysis of heat transfer on nanofluid flow over a linearly porous stretching sheet via Runge Kutta Fehlberg method with shooting technique" and examined the candidate during open defence. This is, therefore, to certify that the thesis is accepted for partial fulfilment of the requirement of the degree of Master in Mathematic Specialization in Numerical analysis.

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Finally approval and acceptance of the thesis is contingent upon submission of its final copy to the Office of Postgraduate Studies (OPGS) through the Department Graduate committee (DGC) and School Graduate Committee (SGC).

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Department Head	Signature	Date
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Office of Postgraduate Studies, Dean	Signature	Date

DEDICATION

I dedicate this project manuscript to my beloved my family, my wife Ganat Ejere , my brother Moges Eshete and my best friend Geleta Tadesse for their sacrifice throughout my education and support for the success of my life.

STATEMENT OF THE AUTHOR

By my signature below, I declare and affirm that thesis is my own work. All sources of materials used for it have been accordingly acknowledged. The thesis has been submitted in partial fulfilment of the requirements for M.Sc. degree at the Salale University and is deposited at the University's Library to be made it available to borrowers under rules of the library. I gravely declare that this project is not submitted to any other institution anywhere for the award of any academic degree, diploma or certificate. Brief quotations from this project are allowable without special permission provided that an accurate acknowledgement of the source is made. Requests for permission for extended quotations from or reproduction of this manuscript in whole or in part may be granted by the head of the department of Mathematics or Dean of the school of Graduate studies when the proposed use of the material is for scholarly interests. In all other instances however, permission must be obtained from the author.

Name: Chalew Eshete Ware Signature: ________________________

Place: Salale Unversity, Salale

Date of submission: June 2022

BIOGRAPHICAL SKETCH

Mr Chalew Eshete Ware was born in Oromia Regional State; north shoa (salale), in 1984G.C.He attended his primary and secondary educations at Kurkura Primary and Dagam Secondary schools, respectively. After completion of his Secondary schools, he joined Hawassa University and graduated with BS degree in Mathematics in July, 2006 G.C. After his graduation, he worked at Giraar Jaarso Wareda Ejersa Kawoo secondary school as a Mathematics teacher. Then, he joined Salale University in September 2013 to pursue a program of study for Master of Science degree in mathematics.

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First of all I would like to thank almighty God who helped me to do every activity in every time. Second, I would like to express my appreciation and sincere gratitude to my advisors kanadmasay Ramasmy (Prof.) and Genanew Gofe (PhD) for their kind encouragement, guidance, support and supervision of my study at Salale university Their constant support and encouragement bolstered my confidence and also I would like to thank members of mathematics department. Imiru Taklu (PhD), Geleta Tadele (PhD) and Getachew Tasfaye (MSc.) for their support in different situation. Lastly, I would like to thank my families and classmates, for their support and encouragement during my study

List of Acronyms and Abbreviation

SYMBOL	MEANING
Cu	Cupper
SWCNTS	Single Wall Carbon Nanotube
ODE	Ordinary Differential Equation
PDE	Partial Differential Equation
CNT	Carbon Nanotube
RKF	Runge Kutta Fehlberg
Re_x	The local Reynolds number
Pr	the Prandtl number
U(x)	Flow velocity of the fluid

Nomenclatures

$(\rho c_p)_{f_n}$	Heat capacitance of the nanofluid
a	a constant,
u, v	velocity components in the x and y directions,
T	Local temperature of the nanofluid,
$\bar{g}$	The acceleration due to gravity,
v_o	The velocity of suction / injection,
K	the permeability of the porous medium,
k_f, k_s	the thermal conductivities of the base fluid and nanoparticle respectively

Greek symbols

ρ_{nf} The effective density of the nanofluid,

μ_{nf} The effective dynamic viscosity of the nanofluid,

α_{nf} The thermal diffusivity of the nanofluid

ζ The solid volume fraction

μ_f The dynamic viscosity of the base fluid,

β_f , β_s the thermal expansion coefficients of the base fluid and nanoparticle, respectively,

ρ_f , ρ_s the densities of the base fluid and nanoparticle respectively,

λ The porous media parameter,

γ The buoyancy parameter,

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ABSTRACT

In this study, we developed the mathematical version and applied the boundary layer approximations to reduce the system of partial differential equations. Similarly, the suitable similarity transformations are implemented at the partial differential equations to make dimensionless system. The dimensionless system solved numerical scheme through Runge Kutta Fehlberg shooting techniques using Maple software. Effects on velocity and temperature distributions for influences of nanoparticle volume fraction, suction parameter, porous parameter, buoyancy parameter and Prandtl number are plotted and physical aspects are discussed. Moreover, outcomes of embedded parameters on skin friction and rate of heat transfer are predicted and given in tabular form. It is found that the temperature of nanofluid and the heat transfer rate enhanced for higher nanoparticle volume fraction. These results are more significant which may apply in the field of engineering and industrial. SWCNTs nanofluids in porous stretching sheet have novel properties that cause them to probably useful in many applications in heat transfer. It is investigated that the thermal boundary layer thickness of the water based Cu nanofluid flow is stronger as correlated to the water based SWCNTs nanofluid flow with increase of buoyancy parameter.

CHAPTER ONE

INTRODUCTION

1.1 Background of the study

A huge range of issues that arise in technological expertise and engineering may be designed via linear or nonlinear differential equations. The glide of fluid over porous media has amazing software in material sciences and engineering collectively with Petroleum industries, filtration and purification of chemical techniques and lots of others. Many fluids are commonly used as warmth transporters in warmness transfer packages. Researchers have constantly laboured on growing advanced warm temperature transfer fluids that have notably higher thermal conductivities than conventionally used fluids.

A nanofluid is a fluid containing nanometre-sized particles, referred to as nanoparticles. The nanoparticles used in nanofluids are typically fabricated from metals, oxides, carbides, or carbon nanotubes. Not unusual base fluids encompass water, ethylene glycol and oil. Nanofluids have novel properties that lead them to potentially beneficial in many programs in warmness switch, consisting of microelectronics, gas cells, pharmaceutical techniques, and hybrid-powered engines, engine cooling/automobile thermal control, home refrigerator, chillier, heat exchanger, in grinding, machining and in boiler flue gas temperature reduction. They exhibit better thermal conductivity and the convective warmness switch coefficient in comparison to the bottom fluid. Understanding of the rheological behaviour of nanofluids is observed to be essential in deciding their suitability for convective warmness switch programs. Nan fluids are commonly used for his or her superior thermal residences as coolants in warmness transfer gadget consisting of heat exchangers, digital cooling device (together with flat plate) and radiators. Warmth transfer over flat plate has been analyzed by using many researchers. However, they're additionally beneficial for their controlled optical houses. Graphene based nanofluid has been determined to beautify Polymerase chain reaction performance. Nanofluids in solar collectors are some other software where nanofluids are hired for his or her tuneable optical properties.

Nanofluids have additionally been explored to beautify thermal desalination technologies, by altering thermal conductivity and soaking up daylight, but floor fouling of the nanofluids poses a major threat to the ones processes. Nanofluid is a new sort of warmness switch medium, containing nanoparticles (one–one hundred nm) which are uniformly and stably distributed in a base fluid. These allotted nanoparticles, usually a metal or metallic oxide substantially enhance the thermal conductivity of the nanofluid, will increase conduction and convection so on Ahmad F., Abdal S., Ayed H., Hussain S., Salim S., &Almatroud A. O. (2021) Shoaib M., Raja M. A. Z., Sabir M. T., Awais M., Islam S., Shah Z., et al. (2021). They display off advanced thermal conductivity and the convective warm temperature switch coefficient compared to the bottom fluid.

Nanofluids in porous stretching sheet have novel houses that lead them to probably useful in masses of applications in warmth switch, which includes microelectronics, gasoline cells, pharmaceutical techniques, and hybrid-powered engines. Nanoparticles are of exquisite clinical hobby as they're efficiently a bridge among bulk materials and atomic or molecular systems. Nanofluids are engineered colloids made from a base fluid and nanoparticles.

Nanofluids have better thermal conductivity and unmarried-segment heat switch coefficients than their base fluids. Convective go along with the waft in porous media has been drastically studied in the state-of-the-art years due to its huge packages in engineering as put up-unintended heat removal in nuclear reactors, solar creditors Roy N. C., & Pop I. (2020) Shoaib M., Raja M. A. Z., Sabir M. T., Islam S., Shah Z., Kumam P., et al. (2020).Drying strategies, warmth exchangers, geothermal and oil healing, constructing manufacturing, and many others. In particular, nanofluid over a porous stretching sheet is an crucial problem in lots of engineering strategies with packages in industries which encompass extrusion, soften-spinning, the new rolling, cord drawing, glass–fibber manufacturing, manufacture of plastic and rubber sheets, cooling of a huge steel plate in a tub, which may be an electrolyte, and so on Al-Hanaya A. M., Sajid F., Abbas N., &Nadeem S. (2020). Lund L. A., Omar Z., Khan I., &Sherif E S. M. (2020)..The only way of regarding a porous medium is as a solid shape with passages thru. Nanofluids may be taken into consideration because the destiny of heat transfer fluids in various heats switches packages. They're anticipated to present higher thermal performance than conventional fluids because of the presence of suspended nanoparticles that have high thermal conductivity. These days, there were numerous investigations that have discovered the enhancement of thermal conductivity and better heat switch rate of nanofluids. Sizable enhancement inside the warmth transfer rate with using numerous nanofluids in diverse application as compared to standard fluids had been said with the aid of numerous researchers. Expertise the homes of nanofluids, which include thermal conductivity, viscosity, and particular warmth, is very critical for the utilization of nanofluids in numerous applications Ishak, A, Nazar, R and pa, I,(2009). Similarly observe of the fundamentals for warmth transfer and friction elements in the case of nanofluids are taken into consideration to be very vital as a way to increase the programs of nanofluids.

In numerous fields of technological knowledge and technology fast progress has counseled the researchers to increase their take a look at toward the regime of boundary layer go with the go with the flow over a stretching sheet. The boundary layer go together with the float conduct toward a linearly or non-linearly stretching sheet plays a huge function for fixing engineering troubles and personal huge programs in manufacturing and production techniques along with steel spinning, rubber sheet production, manufacturing of glass fibres, wire drawing, extrusion of polymer sheets, petroleum industries, polymer processing and so forth. In the ones instances, the final crafted from preferred traits is predicated upon on the charge of cooling inside the technique and the approach of stretching. The dynamics of the boundary layer fluid drift over a stretching and he examined the incompressible constant boundary layer fluid waft as a result of stretching sheet which movements in its very personal plane with linear pace due to the uniform strain programs Ishak, A, Nazar, R and pop, I,(2009) Vajravelu, ok, Prasad, OK.V., Jinho Lee, Changoon Lee, Pop, I, Robert A. Van Gorder,(2011)

Consequences of warmth switch on mixed convection go with the flow inside the presence of suction/injection had been studied with the aid of many authors in one-of-a-kind conditions. But to this point no strive has been made to analyse the effects of nanoparticle volume fraction on Cu, SWCNTs nanofluid go with the flow past a vertical porous stretching sheet and subsequently we've were given taken into consideration the hassle of this kind. The partial differential equations governing the problem under attention are transformed right into a tool of ordinary differential equations which are solved numerically using runge Kutta Felhberg approach with taking pictures method. The fluid is water based totally completely nanofluid containing specific varieties of nanoparticles: Cu and SWCNTs. It is considered that the lowest fluid and the nanoparticles are in thermal equilibrium and no slip takes place among them. The belief might be drawn that the float discipline and temperature profiles are considerably encouraged by means of manner of nanoparticle quantity fraction and distinct parameters.

1.2 Statement of the problem

Based on the applications of nanofluids, large number of theoretical studies of nanofluids has been done in different situation. Boundary layer flow of nanofluid over a stretching sheet was widely investigated Buongiorno, J. and Hu, W.(2005). Due to the literature survey in this paper, it is found that there is a gap found in nanofluids dynamics, such as, the effect of different governing parameters on the flow of nanofluid past vertical stretching geometry; the characteristics of different nanofluids during flow through porous media; particle concentration and type of base fluid. In this analyze, it is attempted to reply the following research questions.

- What are the impacts of different governing parameters on the flow of Cu, SWCNTs nanofluids over a porous vertical stretching sheet?
- What are the proper ratios of SWCNTS and Cu nanoparticles to base fluid to be used to obtain stable nanofluid flow with highest heat transfer?
- What is the effect of steady, temperature jump, boundary conditions, particle volume fraction, and type of base fluid on the stability and flow of nanofluids?

1.3 Objective

1.3.1 General objective

The general objective of this research is to analyze the heat transfer of nanofluid over a linearly porous vertical stretching sheet. Moreover, the effects of nanoparticle volume fraction, buoyancy force and suction / injection and Navier Stokes versions were used in the work.

1.3.2 Specific Objective

The specific objectives of this study are the following:

- To implement Runge Kutta Fehlberg technique with shooting method and develop their Maple programs which solve the boundary layer flow problems to examine the effect of nanoparticle volume fraction on heat transfer over a porous stretching sheet.
- To illustrate velocity and temperature profiles with solution graphically their boundary layer thickness for various governing parameters.

- To provide the effect of different governing parameters on skin friction and rate of heat transfer of practical interest.

1.4 Scope of the study

In this analysis, the problem of two-dimensional and steady laminar boundary layer flow of Cu and SWCNTs nanofluid flow over a linearly porous stretching sheet embedded in a porous media are taken into consideration. Newtonian fluid as base fluid and SWCNTs and Cu nanoparticles may have utilized in the analysis. Mixed convective heat transfer has been investigated with Navier Stokes Model. The effect of thermo physical Ishak, A, Nazar, R and Pop, I,(2009) Vajravelu, K, Prasad, K.V., Jinho Lee, Changoon Lee, Pop, I, Robert A. Van Gorder,(2011)parameters which includes nanoparticle volume fraction, different shape of the nanoparticles, porosity and suction/injection are taken into implementation. The problems can be used numerically employing Runge Kutta Fehlberg with shooting method through the help of Maple software.

1.5 Significance of the study

A large number of researches have been finished on preparation and design of nanofluids flow. However, there is scarce information on nanofluids over a porous stretching sheet.

Therefore, the study may have the following application:

- The numerical algorithms and their Maple programs are improved for the solution techniques may help other researchers to carry out their research.
- The outputs of research are filled the space found in the study area and it is recommended for future study.
- The materials produced by this work have been utilized as a reference for other similar research.

1.6 outline of the thesis

This thesis paper is divided into six chapters. The first chapter is the introduction which introduces the back ground of the study, that is, heat transfer of nanofluids and some important concepts which are essential in the study. Also, some important topics of the study such as statement of the problem, objective of the study, scope of the study, and significance of the study are specified in the first chapter.

In Chapter 2, some important description of fluid flow such as boundary layer flow, mode of heat transfer, governing equations of fluid flow, basic concepts of nanofluids, and some thermo physical properties affecting fluids flow are highlighted.

In Chapter 3, the basic ideas of similarity transformation, and the detailed description of the numerical methods which are used in the thesis namely, Runge Kutta Fehlberg with shooting technique are given.

In Chapter 4, the problem of steady two-dimensional boundary layer flow of Cu and SWCNTs nanofluids past a linearly porous vertical stretching sheet effect was studied. Using suitable similarity transformations, the governing equations of flow are transformed into a system of nonlinear ordinary differential equations and then it is solved numerically employing Runge Kutta Fehlberg with shooting method.

In Chapter 5, the water based Cu and SWCNTs nanofluid flow over a linearly porous vertical stretching sheet was analyzed. The effects of governing parameters such as steadiness, nanoparticle volume fraction, thermal conductivity, viscous dissipation and convective boundary condition are examined. Introducing appropriate similarity variables, the governing equations of fluid flow are solved numerically using RKF with shooting method.

In Chapter 6, finally, general conclusion of the study was provided. Furthermore, future direction of the research work was also recommended. The next section includes all the references that are cited in the thesis. At the end, appendices which consist of the published papers are included.

CHAPTER TWO

2 LITERATURE REVIEW

2.1 Nanofluid and its applications

Nanofluids are dilute liquid suspensions of nanoparticles with at least one of their principal dimensions smaller than 100 nm. Nanofluids in porous stretching sheet have novel properties that cause them to probably useful in many applications in heat transfer. Nanofluids are engineered colloids manufactured from a base fluid and nanoparticles. nanofluids have been found to possess enhanced thermo physical properties such as thermal conductivity, thermal diffusivity, viscosity and convective heat transfer coefficients compared to those of base fluids like oil or water. From the current review, it can be seen that nanofluids clearly exhibit enhanced thermal conductivity, which goes up with increasing volumetric fraction of nanoparticles. The effects of several important factors such as particle size and shapes, clustering of particles, temperature of the fluid, and dissociation of surfactant on the effective thermal conductivity of nanofluids have not been studied adequately. It is important to do more research so as to ascertain the effects of these factors on the thermal conductivity of wide range of nanofluids. Nanoparticles are of fantastic scientific interest as they may be effectively a bridge between bulk substances and atomic or molecular systems. Nan fluids are engineered colloids manufactured from a base fluid and nanoparticles. Nanofluids have higher thermal conductivity and single-phase warmth transfer coefficients than their base fluids Alhussain Z. A., &Tassaddiq A. (2021).Shoaib M., Raja M. A. Sabir M. T., Awais M., Islam S., Shah Z., et al. (2021). Roy N. C.,& Pop I. (2020). Shoaib M., Raja M. A. Z., Sabir MAl-Hanaya A. M., Sajid F., Abbas N., &Nadeem S. (2020)..Lund L. A., Omar Z., Khan I., &Sherif E. S. M. (2020) Kumar K. A., Reddy J. R., Sugunamma V., &Sandeep N. (2018) Kumar K. A., Reddy J. R., Sugunamma V., &Sandeep N. (2018)VenkataRamudu A. C., Anantha Kumar K., Sugunamma V., &Sandeep N. (2020).Kumar K. A., Sugunamma V., Sandeep N., & Mustafa M. T. (2019Al-Hanaya A. M., Sajid F., Abbas N., &Nadeem S. (2020)..Lund L. A., Omar Z., Khan I., &Sherif E. S. M. (2020) Kumar K. A., Reddy J. R., Sugunamma V., &Sandeep N. (2018) Kumar K. A., Reddy J. R., Sugunamma V., &Sandeep N. (2018)VenkataRamudu A. C., Anantha Kumar K., Sugunamma V., &Sandeep N. (2020).Kumar K. A., Sugunamma V., Sandeep N., & Mustafa M. T. (2019. T., Islam S., Shah Z.,Kumam P., et al. (2020). . Convective go with the flow in porous media has been extensively studied inside the recent years because of its extensive programs in engineering as post-accidental heat elimination in nuclear reactors, solar collectors, drying strategies, warmth exchangers, geothermal and oil healing, building construction, and many others. nanofluids have applications in: transportation, heat transfer intensification, electronics applications, industrial cooling applications, heating buildings and reducing pollution, nuclear cooling, energy storage, solar absorption, friction reduction, magnetic sealing, biomedical application, nano drug delivery etc.

Nanofluids are used for cooling of microchips in computer systems and someplace else. They're extensively utilized in other electronic programs which use microfluidics packages. The nanofluid containing magnetic nanoparticles additionally acts as a remarkable-paramagnetic fluid which in an alternating electromagnetic field absorbs strength generating a controllable hyperthermia. The use of nanofluids as coolants would allow for smaller size and higher positioning of the radiators. Nanofluids are suspensions of submicronic solid debris (nanoparticles) in not unusual fluids, Birkoff, G. (1977 Birkoff, G.(1960) Moran, M.J., and Gaggioli, R.A.(1968)The characteristic characteristic of nanofluids is thermal conductivity enhancement, a phenomenon determined by means of Moran, M.J. and R.A. Gaggioli, R.A.(1968) Ibrahim, F.S. and Hamad, M.A.A.(2006) Yurusoy, M and Pakdemirli, M.(2006) Yurusoy, M and Pakdemirli, M.(1999)] Yurusoy, M., Pakdemirli, M and Noyan, O.F.(2001) Hassanien, I.A. and Hamad, M.A.A.(2008) This phenomenon suggests the possibility of the usage of nanofluids in advanced nuclear systems Oztop, H.F and Abu-Nada, E(2008) Akira Nakayama and Hitoshi Koyama,(1989).

2 .2 Carbon nanotube and its applications

A carbon nanotube (CNT) is a tube made of carbon with diameters typically measured in nanometres. Unmarried-wall carbon nanotubes (SWCNTs) unmarried-wall carbon nanotubes are one of the allotropes of carbon, intermediate between fullerene cages and flat grapheme, with diameters in the variety of a nanometre. Despite the fact that not made this manner, single-wall carbon nanotubes may be idealized as cut outs from a -dimensional hexagonal lattice of carbon atoms rolled up alongside one of the Brava is lattice vectors of the hexagonal lattice to form a hole cylinder. In this production, periodic boundary situations are imposed over the period of this roll-up vector to yield a helical lattice of seamlessly bonded carbon atoms on the cylinder surface.

Carbon nanotubes can showcase extremely good electric conductivity, Al-Hanaya A. M., Sajid F., Abbas N., &Nadeem S. (2020) whilst others are semiconductors. They also have brilliant tensile electricity and thermal conductivity because of their nanostructure and electricity of the bonds between carbon atoms. Similarly, they may be chemically modified. These homes are anticipated to be precious in many regions of technology, together with electronics, optics, composite substances (changing or complementing carbon fibers), nanotechnology, and different packages of substances science. Carbon Nanotubes programs CNTs subject emission, CNTs thermal conductivity, CNTs strength storage CNTs conductive properties, CNTs conductive adhesive, CNTs thermal materials Molecular, CNTs structural applications, CNTs fibers and fabric, CNTs biomedical packages and CNTs Air & Water Filtration.

2.3 Heat transfer and its application

Heat is the form of electricity that can be transferred from one machine to another due to a temperature difference or gradient. The technological know-how which offers with the prices of such strength transfer is known as "warmness switch"Kuznetsov, A.V. And D.A. Nield, D.A.(2010). Warmness is a vector amount, flowing in the route of decreasing temp, with a terrible temperature gradient. There are essentially 3 modes of warmth transfer: conduction, convection and radian.

Conduction refers to the thermal power switch in order to occur across a medium, which can be a stable or a fluid due to a temperature difference. Bodily mechanisms for exclusive modes of heat switch conduction. Thermal conduction is the transfer of thermal electricity from the high energetic to the low lively debris of a stationary medium (solids, drinks or gas) due to interactions among the particles. In solids, conduction may be attributed to atomic interest in the form of lattice vibrations and electricity transport by way of the loose electrons. In fluids, conduction takes place because of the collisions and diffusion of the molecules in the course of their random motion. It's miles a microscopic form of heat transfer Heat is transfer from the strong floor to the fluid layer that is in contact with it by using conduction. Then to the adjoining layers warmth will transfer via the random molecular movement. Because of the fluid-surface interplay, a hydrodynamic boundary layer is advanced with 0 pace at the wall and a finite pace stage at the boundary. The increases in powerful thermal conductivity are essential in enhancing the warmth switch behaviour of fluids. Some of other variables additionally play key roles. As an instance, the heat transfer coefficient for forced convection in tubes depends on many physical quantities associated with the fluid or the geometry of the machine via which the fluid is flowing. These portions consist of intrinsic residences of the fluid such as its thermal conductivity, unique warmth, density, and viscosity, in conjunction with extrinsic machine parameters which include tube diameter and period and common fluid pace. Therefore, it's miles vital to measure the warmth transfer overall performance of nanofluids at once beneath glide conditions Very recently Shoaib M., Raja M. A. Z., Sabir M. T., Awais M., Islam S., Shah Z., et al. (2021)Roy N. C., & Pop I. (2020). have tested the affectof nanoparticles on natural convection boundary-layer glide past a vertical plate,Aminossadati, S.M., Ghasemi, B.(2009)Crane, L.J. (1970) Vajravelu, K (1994) Abel, M.S. and Veena, P.H. VAbel, M.S., Khan, S.K. and K.V. Prasad, K.V.(2000).

Convection refers to the thermal power transfer between a surface and a moving fluid when they are at one of a kind temperature ranges. Convection seek advice from the thermal electricity transfer between a solid floor and a moving fluid while they're at exclusive temperature ranges. It entails the combined impact of conduction and fluid motion. Thermal energy transfer by convection is classed as: 1. Natural (loose) convection and forced convection. Forced convection is the switch of thermal strength while the waft is because of external method, inclusive of a fan, a pump or atmospheric winds. Herbal convection is induced with the aid of buoyancy forces due to density variations as a result of temperature differences. There is thermal power convection by means of latent warmness change. This latent warmness is due to change of section from liquid to vapour or vice-versa. Boiling and condensation are examples for such processes. The simple equation for convection warmth switch is referred to as Newton’s law of cooling.

Thermal radiation refers back to the internet switch of thermal strength between surfaces at one-of-a-kind temperature degrees, in the absence of an intervening medium among them. This happens because of electromagnetic waves emitted from a warm frame.

2.4 Buoyancy force and its applications

Buoyancy or up thrust, is an upward pressure exerted with the aid of a fluid that opposes the burden of a partially or absolutely immersed item. In a column of fluid, pressure increases with intensity as a result of the load of the overlying fluid. For that reason the strain at the bottom of a column of fluid is greater than at the top of the column. In addition, the pressure at the bottom of an object submerged in a fluid is extra than on the top of the object. The stress difference effects in a internet upward force at the object Akira Nakayama and Hitoshi

Koyama,(1989). The value of the force is proportional to the stress distinction, and equivalent to the weight of the fluid that might occupy the submerged quantity of the itemAl-Hanaya A. M., Sajid F., Abbas N., &Nadeem S. (2020)..Lund L. A., Omar Z., Khan I., &Sherif E. S. M. (2020) Kumar K. A., Reddy J. R., Sugunamma V., &Sandeep N. (2018) Kumar K. A., Reddy J. R., Sugunamma V., &Sandeep N. (2018)VenkataRamudu A. C., Anantha Kumar K., Sugunamma V., &Sandeep N. (2020).Kumar K. A., Sugunamma V., Sandeep N., & Mustafa M. T. (2019 .Buoyancy finds many applications in our daily existence especially for the duration of our interactions with fluids. Some of the programs are as below:

•The submarines are designed keeping in mind the idea of buoyancy. It could waft on as well as sink in water. That is finished by way of filling the submarine with air thereby organising nice buoyancy which enables it to glide. In addition, it fills in water to establish terrible buoyancy thereby allowing it to sink in water.

•A ship made of iron floats on water while a ball manufactured from iron sinks in it. The ship floats through displacing water same to its weight, thereby getting enough buoyant pressure to go with the flow on water.

•we will swim due to buoyancy. At the same time as being within the swimming pool or pond, we displace water same to our frame weight thereby getting assist from the water itself to go with the flow on it and swim.

•The lifestyles jackets help the person in open water bodies to float on them thereby stopping drowning. Once more, this permits to displace water equal to the weight of the consumer to get sufficient up thrust for floating on water.

2.5 Porous media and its applications

A porous medium is vicinity which contains pores that offers resistance to fluid Flow. The extent of the fluid to flow throughout a porous medium is decided via the Porosity or permeability of the medium, and the primary law governing the drift of fluids thru porous media Abel, M.S. And Veena, (1998). The flow of fluid in Heat storage in beds, biological structures, irrigation troubles, and the go with the flow of water in Aquifers are some examples of fluid glide in porous media. Fluid flow thru porous media has substantial realistic application in fabric sciences And engineering which includes Petroleum industries, seepage of water in river beds, filtration and purification of chemical techniques and lots of others. In porous media, a connected pore network constructs a complicated machine. The trails of fluid/electrical glide aren't any straight due to the unusual pore duration distribution. It may motive that the flowing route may be greater tortuous and meandering for worse connectivity. A key trouble for know-how the delivery phenomenon in porous media is a manner to accurately describe the flowing path of particles in pore location. Tortuosity expresses the statistical motion behaviour of the shipping system in preference to an appropriate length ratio of every particle music. But, tortuosity is sensitive to the spatial heterogeneity of porous media, and with the conventional tortuosity equation with empirical parameters, it is difficult to depict the tortuous characteristics Choi, S. In: D.A. Siginer, H.P. Wang (Eds.) (1995), Masuda, H., Ebata, A.,Teramae, ok. And Hishinuma, N. (1993) Buongiorno, J. And Hu, W.(2005) Buongiorno, J. (2006) Kuznetsov, A.V. And D.A. Nield, D.A. (2010) Nield, D.A. And Kuznetsov, A.V. (2009) Cheng, P. And Minkowycz, W.J. (1977).

Porous media may be characterised at two brilliant degrees: microscopic and macroscopic. On the microscopic degree, the shape is expressed in phrases of a statistical description of the pore period distribution, degree of inter-connection and orientation of the pores, fraction of vain pores, and so forth. In the macroscopic method, bulk parameters are employed which have been averaged over scales plenty massive than the dimensions of pores. The ones tactics are complementary and are used extensively relying upon the objective. Definitely, the microscopic description is crucial for records ground phenomena which includes adsorption of macromolecules from polymer answers and the blockage of pores, and lots of others., whereas the macroscopic approach is frequently quite ok for manner design where fluid go with the flow, warmth and mass transfer are of best interest, and the molecular dimensions are an lousy lot smaller than the pore length. Birkoff, G. (1977) Birkoff, G.(1960) Moran, M.J., and Gaggioli, R.A.(1968) Moran, M.J. And R.A. Gaggioli, R.A.(1968) Ibrahim, F.S. And Hamad, M.A.A.(2006) Yurusoy, M and Pakdemirli, M.(2006) Yurusoy, M and Pakdemirli, M.(1999) Yurusoy, M., Pakdemirli, M and Noyan, O.F.(2001) Hassanien, I.A. And Hamad, M.A.A.(2008) Group theoretic Oztop, H.F and Abu-Nada, E(2008). Akira Nakayama and Hitoshi Koyama,(1989)The glide through porous media occurs in endless quantity of programs starting from geophysical float, restoration of oil, gasoline, minerals from mom earth, delivery and sequestration of contaminant in subsurface, to the massive scale chemical approaches regarding catalyst, filter out, adsorbent, and also modelling of physiological procedures.

CHAPTER THREE

3. RESEARCH METHODOLOGY

3.1 Formulation of the Problem

Consider the steady laminar two- dimensional flow of an incompressible viscous nanofluid past a linearly porous semi-infinite stretching sheet. The temperature at the stretching surface takes the constant value T_w, while the ambient value, attained as y tends to infinity, takes the constant value T_∞.

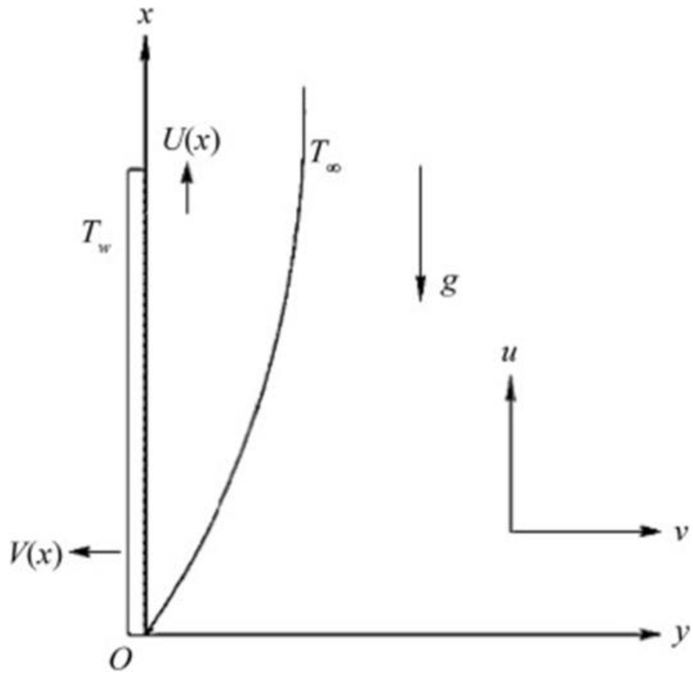


Figure3.1 Flow configuration and coordinate system

The fluid is a water based nanofluid containing different types of nanoparticles: Cu SWCNTs. It is assumed that the base fluid and the nanoparticles are in thermal equilibrium and no slip occurs between them. Under the above assumptions, the boundary layer equations governing the flow and concentration field can be written in dimensional form as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (3.1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho_{nf}}\left[\mu_{nf}\frac{\partial^2 u}{\partial \bar{y}^2}+(\rho\beta)_{nf}\,g(T-T_\infty)-(\frac{\nu_f}{K}\rho_{nf})u\right] \tag{3.2}$$

$$u\frac{\partial T}{\partial \bar{x}}+v\frac{\partial T}{\partial \bar{y}}=\alpha_{nf}\frac{\partial^2 T}{\partial \bar{y}^2} \tag{3.3}$$

The boundary conditions of these equations are

$$u=U=a\,x, v=-v_0, T=T_w \text{ at } \bar{y}=0\,;\, u\to 0, T\to T_{\infty w} \text{ as } \bar{y}\to\infty \tag{3.4}$$

The suffixes w and ∞ denote surface and ambient conditions. a is a constant, u and v are the velocity components in the x and y directions, T is the local temperature of the nanofluid, $\bar{g}$ is the acceleration due to gravity, v_o is the velocity of suction / injection, K is the permeability of the porous medium, ρ_{nf} is the effective density of the nanofluid, μ_{nf} is the effective dynamic viscosity of the nanofluid, α_{nf} is the thermal diffusivity of the nanofluid which are defined as (see Aminossadati and Ghasemi),

$$\rho_{nf}=(1-\zeta)\rho_f+\zeta\rho_s,\;\; \mu_{nf}=\frac{\mu_f}{(1-\zeta)^{2.5}},\; (\rho c_p)_{nf}=(1-\zeta)(\rho c_p)_f+\zeta(\rho c_p)_s$$

$$(\rho\beta)_{nf}=(1-\zeta)(\rho\beta)_f+\zeta(\rho\beta)_s,\; \alpha_{nf}=\frac{k_{f_n}}{(\rho\, c_p)_{f_n}},$$

Here ζ is the solid volume fraction, μ_f is the dynamic viscosity of the base fluid, β_f and β_s are the thermal expansion coefficients of the base fluid and nanoparticle, respectively, ρ_f and ρ_s are the densities of the base fluid and nanoparticle, respectively, k_{f_n} is the thermal conductivity of the nanofluid and $(\rho\, c_p)_{f_n}$ is the heat capacitance of the nanofluid, which are defined as

$$(\rho\, c_p)_{f_n}=(1-\zeta)(\rho\, c_p)_f+\zeta(\rho\, c_p)_s$$

$$\frac{k_{nf}}{k_f}=\frac{k_s+2k_f-2\zeta(k_f-k_s)}{k_s+2k_f+2\zeta(k_f-k_s)} \tag{3.5}$$

Where k_f and k_s are the thermal conductivities of the base fluid and nanoparticle respectively.

The stream function rewarding equation (1) with

$$u = \frac{\partial \psi}{\partial y} \text{ and } v = -\frac{\partial \psi}{\partial x} \quad . \tag{3.6}$$

Based on the equation.(3.6) with the similarity variables

$$\eta = y\sqrt{\frac{a}{\nu_f}} \; , \psi = \sqrt{a\nu_f}\; x f(\eta) \tag{3.7}$$

Using the above mentioned similarity transformation, Equations (3.6) and (3.7), we get

$$f''' + A\left((ff'' - (f')^2 - \lambda f' \right) + B\gamma\, \theta = 0 \tag{3.8}$$

$$\theta'' + \frac{\Pr C}{E}\left(f\theta' - f'\theta \right) = 0 \tag{3.9}$$

With boundary conditions

$$f'(0) = 1,\; f(0) = S\,, \theta(0) = 1 \;\; f'(\infty) = 0\,, \theta(\infty) = 0 \tag{3.10}$$

where $A = (1-\varsigma)^{2.5}\left(1-\varsigma+\varsigma\frac{\rho_s}{\rho_f} \right)$, , $B = (1-\varsigma)^{2.5}\left(1-\varsigma+\varsigma\frac{(\rho\beta)_s}{(\rho\beta)_f} \right)$,

$C = (1-\varsigma)^{2.5}\left(1-\varsigma+\varsigma\frac{(\rho c_p)_s}{(\rho c_p)_f} \right)$ and $E = \dfrac{k_{nf}}{kf}$

where $\Pr = \dfrac{\nu_f}{\alpha_f}$ is the Prandtl number, $\lambda = \dfrac{\nu_f}{a\, K}$ is the porous media parameter, $\gamma = \dfrac{g\,(T_w - T_\infty)(\rho\beta)_f}{\rho_f\, a\, U}$ is the buoyancy or natural convection parameter and $\dfrac{v_0}{\sqrt{a\nu_f}} = S$, suction if S > 0 and injection if S < 0 . It is worth mentioning that $\gamma > 0$ aids the flow and $\gamma < 0$ opposes the flow, while $\gamma = 0$ i.e., $(T_w - T_\infty)$ represents the case of forced convection flow. On the other hand, if γ is of a significantly greater order of magnitude than one, then the buoyancy forces will be predominant. Hence, combined convective flow exists when $\gamma = O(1)$.

For practical purposes, the functions $f(\eta)$ and $\theta(\eta)$ allow us to determine the skin friction coefficient

$$C_f = \frac{\mu_{f_n}}{\rho_f U^2}\left(\frac{\partial u}{\partial y}\right)_{at\ y=0} = -\frac{1}{(1-\zeta)^{2.5}}\left(\mathrm{Re}\, x\right)^{-\frac{1}{2}} f''(0) \tag{3.11}$$

and the Nusselt number

$$Nu_x = \frac{x k_{f_n}}{k_f (T_w - T_\infty)}\left(\frac{\partial T}{\partial y}\right)_{at\ y=0} = -\left(\mathrm{Re}\, x\right)^{\frac{1}{2}} \frac{k_{f_n}}{k_f}\theta'(0) \tag{3.12}$$

respectively. Here, $\mathrm{Re}_x = \frac{U\ x}{\nu_f}$ is the local Reynolds number.

CHAPTER FOUR

4 DESCRIPTIONS OF THE PROPOSED METHODS

4.1 Similarity Transform

Governing Equations of the problem

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho_{nf}}\left(\mu_{nf}\frac{\partial^2 u}{\partial y^2}+(\rho\beta)_{nf}g(T-T_\infty)-\frac{\nu_f}{K}\rho_{nf}u\right)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_{nf}\frac{\partial^2 T}{\partial y^2}$$

With boundary conditions are

$$u=0, v=v_0, -k_{nf}\frac{\partial T}{\partial y}_{at\ \ y=0}=q(x) \text{ at } y=0$$

$$u=U, T=T_\infty, \text{ at } y\to\infty$$

Define $\psi = xf(\eta)\sqrt{a\nu_f}=hf(\eta)$ and $\eta=\sqrt{\frac{a}{\nu_f}}\,y=py$, $\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}$

Let $h(x)=x\sqrt{a\nu_f}$ and $p(x)=\sqrt{\frac{a}{\nu_f}}$, then $\psi = xf(\eta)\sqrt{a\nu_f}=hf(\eta)$ and $\eta=\sqrt{\frac{a}{\nu_f}}\,y=py$, .

4.1.1 Momentum equation

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho_{nf}}\left(\mu_{nf}\frac{\partial^2 u}{\partial y^2}+(\rho\beta)_{nf}g(T-T_\infty)\right)-\frac{\nu_f}{K}\rho_{nf}u$$

For similarity transformation, we have $u=\frac{\partial\psi}{\partial y}$ and $v=-\frac{\partial\psi}{\partial x}$, where $\psi=h(x)f(\eta)$ and

$\eta=\sqrt{\frac{a}{\nu_f}}\,y=py$, or $y=\frac{\eta}{p}$, are stream function.

(i) $h'(x)=\sqrt{a\nu_f}$

(ii) $\eta=\sqrt{\dfrac{a}{\nu_f}}\ y=py,\ p^{'}(x)=0$

Hence,

(i) $u=\dfrac{\partial\psi}{\partial y}=\dfrac{\partial}{\partial y}\big(h(x)f(x,\eta)\big)=h(x)\dfrac{\partial f}{\partial\eta}\dfrac{\partial\eta}{\partial y}=h(x)p(x)f^{'}=Uf^{'}$, where $\dfrac{\partial\eta}{\partial y}=p(x)$,

$f^{'}=\dfrac{\partial f}{\partial\eta}$, and $U=hp$.

$$u_x=\frac{\partial u}{\partial x}=\frac{\partial}{\partial x}\Big(h(x)p(x)f^{'}\Big)=\big(hp\big)^{'}f^{'}+(hp)\left[f_x^{'}+f^{''}\frac{p^{'}}{p}\eta\right]$$, where $\dfrac{\partial\eta}{\partial x}=p^{'}y=\dfrac{p^{'}}{p}\eta$.

$$=h^{'}pf^{'}+hp^{'}f^{'}+hpf_x^{'}+hp^{'}f^{''}\eta=af'$$

$$u_y=\frac{\partial u}{\partial y}=\frac{\partial}{\partial y}\Big(h(x)p(x)f^{'}\Big)=h(x)p(x)\frac{\partial f^{'}}{\partial\eta}\frac{\partial\eta}{\partial y}=h(x)p(x)f^{''}p(x)=hp^2f^{''}$$

$$u_{yy}=\frac{\partial^2u}{\partial y^2}=\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)=\frac{\partial}{\partial y}\Big(hp^2f^{''}\Big)=hp^2f^{'''}p=hp^3f^{'''}$$

(ii) $v=-\dfrac{\partial\psi}{\partial x}=-\dfrac{\partial}{\partial x}\big(h(x)f(x,\eta)\big)=-\left[h^{'}f+h\left(f_x+f^{'}\dfrac{\partial\eta}{\partial x}\right)\right]$

$=-\left[h^{'}f+h\left(f_x+f^{'}p^{'}\dfrac{\eta}{p}\right)\right]=-\left\lfloor f\sqrt{a\nu_f}\right\rfloor$, where $f_x=\dfrac{\partial f}{\partial x}$.

$$\frac{\partial v}{\partial y}=-\sqrt{a\nu_f}\frac{\partial f}{\partial y}p=-\sqrt{a\nu_f}f'p=-af'$$

(iii) $u\dfrac{\partial u}{\partial x}=hpf^{'}\left[\big(hp\big)^{'}f^{'}+hpf_x^{'}+hp^{'}f^{''}\eta\right]$ and

$$v\frac{\partial u}{\partial y}=-\left[h^{'}f+h\left(f_x+\frac{p^{'}}{p}\eta f^{'}\right)\right]hp^2f^{''}$$

Furthermore, the momentum equation

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho_{nf}}\left(\mu_{nf}\frac{\partial^2u}{\partial y^2}+\big(\rho\beta\big)_{nf}g(T-T_\infty)\right)-\frac{\nu_f}{K}\rho_{nf}u$$

Becomes

$$hpf^{'}\left[h^{'}pf^{'}+hp^{'}f^{'}+hpf_x^{'}+hp^{'}f^{''}\eta\right]-hp^2f^{''}\left[h^{'}f+h\left(f_x+\frac{p^{'}}{p}\eta f^{'}\right)\right]$$

$$\nu_{nf}hp^3f'''+\frac{(\rho\beta)_{nf}g(T-T_\infty)}{\rho_{nf}}-\frac{1}{\rho_{nf}}\frac{\nu_f}{K}\rho_{nf}phf'$$

$$hpf^{'}[af']-f\sqrt{a\nu_f}\,p^2hf''=\frac{1}{\rho_{nf}}\left(\mu_{nf}p^3hf'''+(\rho\beta)_{nf}g(T_w-T_\infty)\theta-\frac{\nu_f}{K}\rho_{nf}phf'\right)$$

$$hpf^{'}[af']-f\sqrt{a\nu_f}\,p^2hf''=\nu_{nf}p^3hf'''+\frac{1}{\rho_{nf}}(\rho\beta)_{nf}g(T_w-T_\infty)\theta-\frac{1}{\rho_{nf}}\frac{\nu_f}{K}\rho_{nf}phf'$$

The last equation is divided by $\nu_{nf}hp^3$, we obtain

$$hpf^{'}[af']-f\sqrt{a\nu_f}\,p^2hf''=\frac{1}{\rho_{nf}}\left(\mu_{nf}p^3hf'''+(\rho\beta)_{nf}g(T_w-T_\infty)\theta-\frac{\nu_f}{K}\rho_{nf}phf'\right)$$

$$\frac{phaf'}{\nu_{nf}hp^3}-\frac{hp^2f''}{\nu_{nf}hp^3}\left[f\sqrt{a\nu_f}\right]=f'''+\frac{1}{\nu_{nf}hp^3\rho_{nf}}(\rho\beta)_{nf}g(T_w-T_\infty)\theta-\frac{\nu_f}{\nu_{nf}hp^3K}phf'$$

(i) $\dfrac{phaf'}{\nu_{nf}hp^3}=(f')^2\dfrac{\nu_f}{\nu_{nf}}=A(f')^2$ where

$$\frac{\nu_f}{\nu_{nf}}=\frac{\rho_{nf}}{\mu_{nf}}\nu_f=(1-\varsigma)^{2.5}\left(1-\varsigma+\varsigma\frac{\rho_s}{\rho_f}\right)\frac{\nu_f}{\nu_f}=(1-\varsigma)^{2.5}\left(1-\varsigma+\varsigma\frac{\rho_s}{\rho_f}\right)=A$$

(ii) $\dfrac{hp^2f''}{\nu_{nf}hp^3}\left[f\sqrt{a\nu_f}\right]=-Aff''$

(iii)

$$\frac{1}{\nu_{nf}hp^3\rho_{nf}}(\rho\beta)_{nf}g(T_w-T_\infty)\theta$$

$$=B\frac{(\rho\beta)_f g(T_w-T_\infty)}{aU\rho_f}\theta=B\gamma\,\theta \qquad where\ B=(1-\varsigma)^{2.5}\left(1-\varsigma+\varsigma\frac{(\rho\beta)_s}{(\rho\beta)_f}\right)$$

$\dfrac{(\rho\beta)_f g(T_w-T_\infty)}{aU\rho_f}=\gamma$ -Buoyancy parameter

(iv) $\dfrac{\nu_f}{\nu_{nf}hp^3K}phf'=A\lambda f'$ where $\dfrac{\nu_f}{aK}=\lambda$ porous parameter

Therefore the given equation becomes

$$\frac{phaf'}{\nu_{nf}hp^3}-\frac{hp^2 f''}{\nu_{nf}hp^3}\left[f\sqrt{a\nu_f}\right]= f''' + \frac{1}{\nu_{nf}hp^3\rho_{nf}}\left(\rho\beta\right)_{nf} g(T_w - T_\infty)\theta - \frac{\nu_f}{\nu_{nf}hp^3K} phf'$$

Hence

$$f''' + A\left((ff'' - (f')^2 - \lambda f'\right) + b\gamma\,\theta = 0$$

4.1.2 Energy Equation

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_{nf}\frac{\partial^2 T}{\partial y^2}$$

Let $\theta(\eta) = \dfrac{T - T_\infty}{T_w - T_\infty}$, $\Pr = \dfrac{\mu_f\,(c_p)_f}{k_f} = \dfrac{\nu_f}{\alpha_f}$.

We define non-dimensional parameter $\theta(\eta) = \dfrac{T - T_\infty}{T_w - T_\infty}$, and

$$T - T_\infty = \theta\,\Delta T$$

$$T_x = \frac{\partial T}{\partial x} = \frac{\partial\left(T_\infty + \theta\Delta T\right)}{\partial x} = \theta + \Delta T\theta'\,0 = \theta$$

$T_x = \theta$, where $\theta^{'} = \dfrac{\partial\theta}{\partial\eta}$, $\dfrac{\partial\eta}{\partial x} = 0y = 0$

$T_y = \dfrac{\partial\left(T_\infty + \theta\,\Delta T\right)}{\partial y} == \Delta T\theta^{'}p$, where $\dfrac{\partial\eta}{\partial y} = p(x)$.

$T_{yy} = \dfrac{\partial^2\left(T_\infty + \theta\,\Delta T\right)}{\partial y^2} = \dfrac{\partial}{\partial y}\left(\Delta T\theta^{'}p\right) = \Delta T\theta^{''}p^2$.

From previous results, $v == -\left\lfloor f\sqrt{a\nu_f}\right\rfloor$ and $u = hpf^{'}$, then

Therefore,

(i) $\alpha_{nf}\dfrac{\partial^2 T}{\partial y^2} = \alpha_{nf}\Delta T\theta^{''}p^2$

Furthermore, the energy equation $u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_{nf}\frac{\partial^2 T}{\partial y^2}$ becomes

$$phf'(\theta)-\sqrt{a\nu_f}\, f\ \Delta T\ \theta' p=\alpha_{nf}\Delta T p^2\theta''$$

The last equation is divided by $\alpha_{nf}\Delta T p^2$, we get

$$\frac{phf'}{\alpha_{nf}\Delta T p^2}(\theta)-\frac{\sqrt{a\nu_f}\, f}{\alpha_{nf}\Delta T p^2}\Delta T\ \theta' p=\theta'' \text{ Where } \alpha_{nf}=\frac{k_{nf}}{(\rho c_p)_{nf}}$$

$$\frac{phf'}{\alpha_{nf}\Delta T p^2} \text{ where } \alpha_{nf}=\frac{C}{\Pr E} \text{ and } E=\frac{k_{nf}}{kf},\ C=(1-\varsigma)^{2.5}\left(1-\varsigma+\varsigma\frac{(\rho c_p)_s}{(\rho c_p)_f}\right)$$

$$\frac{phf'\,\theta}{\alpha_{nf}p^2}=\frac{C}{\Pr E}f'\theta$$

$$\frac{\sqrt{a\nu_f}\, f}{\alpha_{nf}\Delta T p^2}\Delta T\ \theta' p=\frac{\Pr C}{E}f\theta'$$

Hence the energy equation becomes

$$\theta''+\frac{\Pr C}{E}\left(f\theta'-f'\theta\right)=0$$

Where $E=\frac{k_{nf}}{kf}$, $C=(1-\varsigma)^{2.5}\left(1-\varsigma+\varsigma\frac{(\rho c_p)_s}{(\rho c_p)_f}\right)$

4.1.3 Boundary conditions

At $y=0$: $u=U, v=-v_0$, $T=T_w$ then $\eta=py=0$ and

$$u=Uf' \Rightarrow U=Uf' \Rightarrow \frac{\partial f}{\partial\eta}=1, f'(0)=1.$$

$$v=v_0 \Rightarrow v=-\left[h' f+h\left(f_x+\frac{p'}{p}\eta f'\right)\right]=-\sqrt{a\nu_f}\, f(0)=-v_0$$

$$f(0)=\frac{v_0}{\sqrt{a\nu_f}}=S, Suction/Injection; Suction\ if\ S \succ 0; Injection\ if\ S \prec 0$$

$$\theta(0)=\frac{T_w-T_\infty}{T_w-T_\infty}=1$$

$u=Uf^{'}=0$ as $y\to\infty$

$$Uf^{'}(\infty)=0 \;\; f^{'}(\infty)=0$$

$\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}$; $T\to T_\infty$ as $y\to\infty$

$$\theta(\infty)=\frac{T_\infty-T_\infty}{T_w-T_\infty}=0$$

Finally we have

$$f^{'''}+A\left((ff''-(f')^2-\lambda f'\right)+B\gamma\,\theta=0 \tag{4.1}$$

$$\theta^{''}+\frac{\Pr C}{E}\left(f\theta'-f'\theta\right)=0 \tag{4.2}$$

With boundary conditions

$$f'(0)=1,\; f(0)=S\,,\theta(0)=1,\; f^{'}(\infty)=0\,,\theta(\infty)=0 \tag{4.3}$$

where $A=(1-\varsigma)^{2.5}\left(1-\varsigma+\varsigma\frac{\rho_s}{\rho_f}\right)$, , $B=(1-\varsigma)^{2.5}\left(1-\varsigma+\varsigma\frac{(\rho\beta)_s}{(\rho\beta)_f}\right)$,

and $E=\frac{k_{nf}}{kf}$

$$\frac{v_0}{\sqrt{a\nu_f}}=S, Suction/Injection; Suction\;if\;\; S\;\phi\;0; Injection\;if\;\; S\;\pi\;0$$

$\frac{\nu_f}{aK}=\lambda$ - Porous parameter, .. $\frac{(\rho\beta)_f\,g(T_w-T_\infty)}{aU\rho_f}=\gamma$ - Buoyancy parameter

4.2. Integration using Run.ge-Kutta-Fehlberg method

When the governing equations (4.1) and (4.2) with the following initial conditions (4.3) have been known, then the initial value problems can be solved using Runge-Kutta-Fehlberg with shooting method available in Maple software

By defining new variables as $f = f(\eta), u(\eta) = f'(\eta), v(\eta) = f''(\eta), w(\eta) = \theta'(\eta)$

The system of first order differential equations with boundary conditions are

$$f'(\eta) = u(\eta)$$

$$u'(\eta) = v(\eta)$$

$$v'(\eta) + A\big((f(\eta)v(\eta) - (u(\eta))^2 - \lambda u(\eta)\big) + B\gamma\theta(\eta) = 0$$

$$\theta'(\eta) = w(\eta)$$

$$w'(\eta) + \frac{\Pr C}{E}\big(f(\eta)w(\eta)\big) - u(\eta)\theta(\eta)) = 0$$

With boundary conditions

$$u(0) = 1,\ f(0) = S,\ \theta(0) = 1,\ u(\infty) = 0,\ \theta(\infty) = 0$$

Where $A = (1-\varsigma)^{2.5}\left(1-\varsigma+\varsigma\frac{\rho_s}{\rho_f}\right),\ ,\ B = (1-\varsigma)^{2.5}\left(1-\varsigma+\varsigma\frac{(\rho\beta)_s}{(\rho\beta)_f}\right),$

$$C = (1-\varsigma)^{2.5}\left(1-\varsigma+\varsigma\frac{(\rho c_p)_s}{(\rho c_p)_f}\right) \text{ and } E = \frac{k_{nf}}{kf}$$

$$\frac{v_0}{\sqrt{a\nu_f}} = S, Suction/Injection; Suction\ if\ S \ \phi\ 0; Injection\ if\ S\ \pi\ 0$$

$\frac{\nu_f}{aK} = \lambda$ - Porous parameter, $\frac{(\rho\beta)_f g(T_w - T_\infty)}{aU\rho_f} = \gamma$ - Buoyancy parameter

4.2.1 Numerical Analysis

Equations (4.1) and (4.2) subjected to the boundary condition (4.3)) are converted into the following simultaneous system of first order differential equations as follows:

$$A = (1-\varsigma)^{2.5}\left(1-\varsigma+\varsigma\frac{\rho_s}{\rho_f}\right),\ ,\ B = (1-\varsigma)^{2.5}\left(1-\varsigma+\varsigma\frac{(\rho\beta)_s}{(\rho\beta)_f}\right),$$

$$C = (1-\varsigma)^{2.5}\left(1-\varsigma+\varsigma\frac{(\rho c_p)_s}{(\rho c_p)_f}\right) \text{ and } E = \frac{k_{nf}}{kf}$$

$$f'(\eta) = u(\eta)$$

$$u'(\eta) = v(\eta)$$

$$v'(\eta) + A\big((f(\eta)v(\eta) - (u(\eta))^2 - \lambda u(\eta)\big) + B\gamma\theta(\eta) = 0$$

$$\theta'(\eta) = w(\eta)$$

$$w'(\eta) + \frac{\Pr C}{E}\big(f(\eta)w(\eta)\big) - u(\eta)\theta(\eta)) = 0$$

with boundary conditions

$$u(0) = 1,\ f(0) = S,\ \theta(0) = 1, u(\infty) = 0, \theta(\infty) = 0, v(0) = \alpha, \theta(0) = \beta.$$

Where α and β are priori unknowns to be determined as a part of the solution.

This software uses Runge–Kutta–Fehlberg method with shooting technique as default to solve the boundary value problems numerically using the Dsolve command MAPLE 18. The values of α and β are determined upon solving the boundary conditions $v(0) = \alpha, \theta(0) = \beta$ with trial and error basis. The numerical results are represented in the form of the dimensionless velocity and temperature in the presence of water based SWCNTs and Cu.

CHAPTER FIVE

5 RESULTS AND DISCUSSION

Equations (4.1) and (4.2) subject to the boundary conditions (4.3) have been solved numerically for some values of the governing parameters $\Pr, \zeta, n, \lambda$ *and* M using Runge Kutta Fehlberg method with shooting technique. Table 5.2 thermo physical properties of fluid and nanoparticles. Obtained by Grubka and BobbaAli ,Ishak et al. and Vajravelu et al. for different values of $\Pr$ in Table 5.1. We notice that the comparison shows good agreement for each value of $\Pr$. Therefore, we are confident that the present results are very accurate.

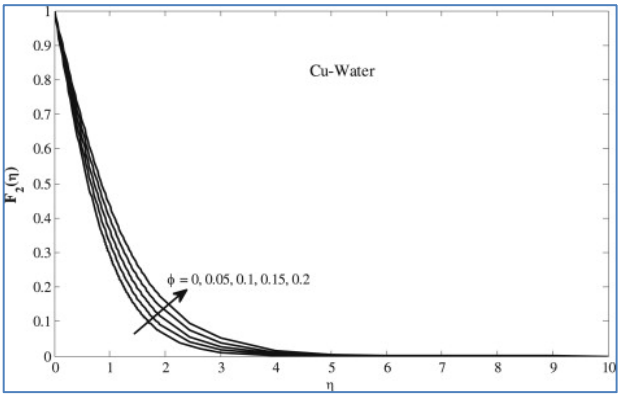
Table 5. 1 validate results for $-\theta'(0)$ with previous published works

Pr	*Grubka and Bobba* 1994)	*Ishak et al.* (2009)	Present work $\zeta = 0, M = n = S = \lambda = 0$
0.72	0.8086	0.8086	0.80867991
1.00	1.0000	1.0000	1.00000000
3.00	1.9237	1.9237	1.92368891
7.00	3.7207	3.7207	3.72077927
10.0	12.294	12.2941	12.2941328

Table 5.1 thermo physical properties of nanofluid and particles

	$\rho(kg/m^3)$	$c_p(J/kgK)$	$k(W/mK)$	$\beta \times 10^5 (K^{-1})$
Pure water $(\zeta = 0.0)$	997.1	4179	0.613	21
Copper $(Cu)(\zeta = 0.05)$	8933	385	401	1.67
SWCNTs $(\zeta = 0.01)$	2600	425	6600	0.99

In order to ascertain the accuracy of our numerical results, the present study is compared with the available exact solution in the literature. The temperature profiles for Prandtl number $\Pr$ are compared with the available exact solution of Hamad et al(2021).Is shown in Figs.5.1 & 5.2. It is observed that the agreements with the theoretical solution of temperature profiles are excellent. For a given ζ , it is clear that there is a fall in temperature with increasing the nanoparticle volume fraction parameter. This is due to the fact that there would be a decrease of thermal boundary layer thickness with the increase of nanoparticle volume fraction parameter as one can see from Figs.5.1&5.2 by comparing the curves with $\zeta = 0.05, 0.10 \; and \; 0.15$.



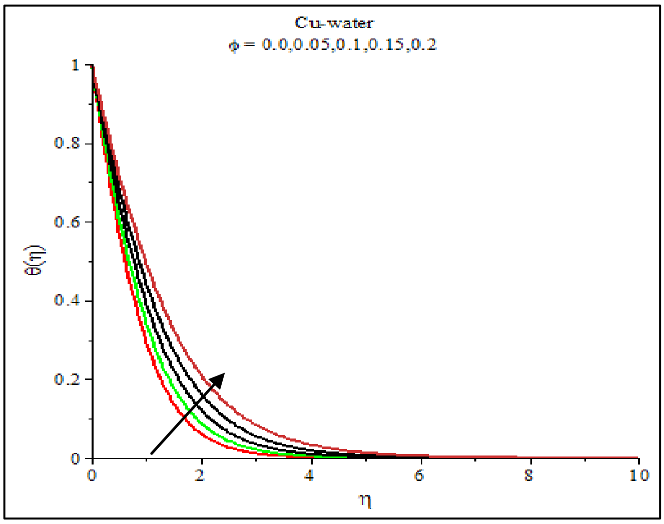


Figure 5.2 *Comparison of the temperature profiles (present result) with Hamad et al.* $pr = 6.2, = \lambda = n = 0$

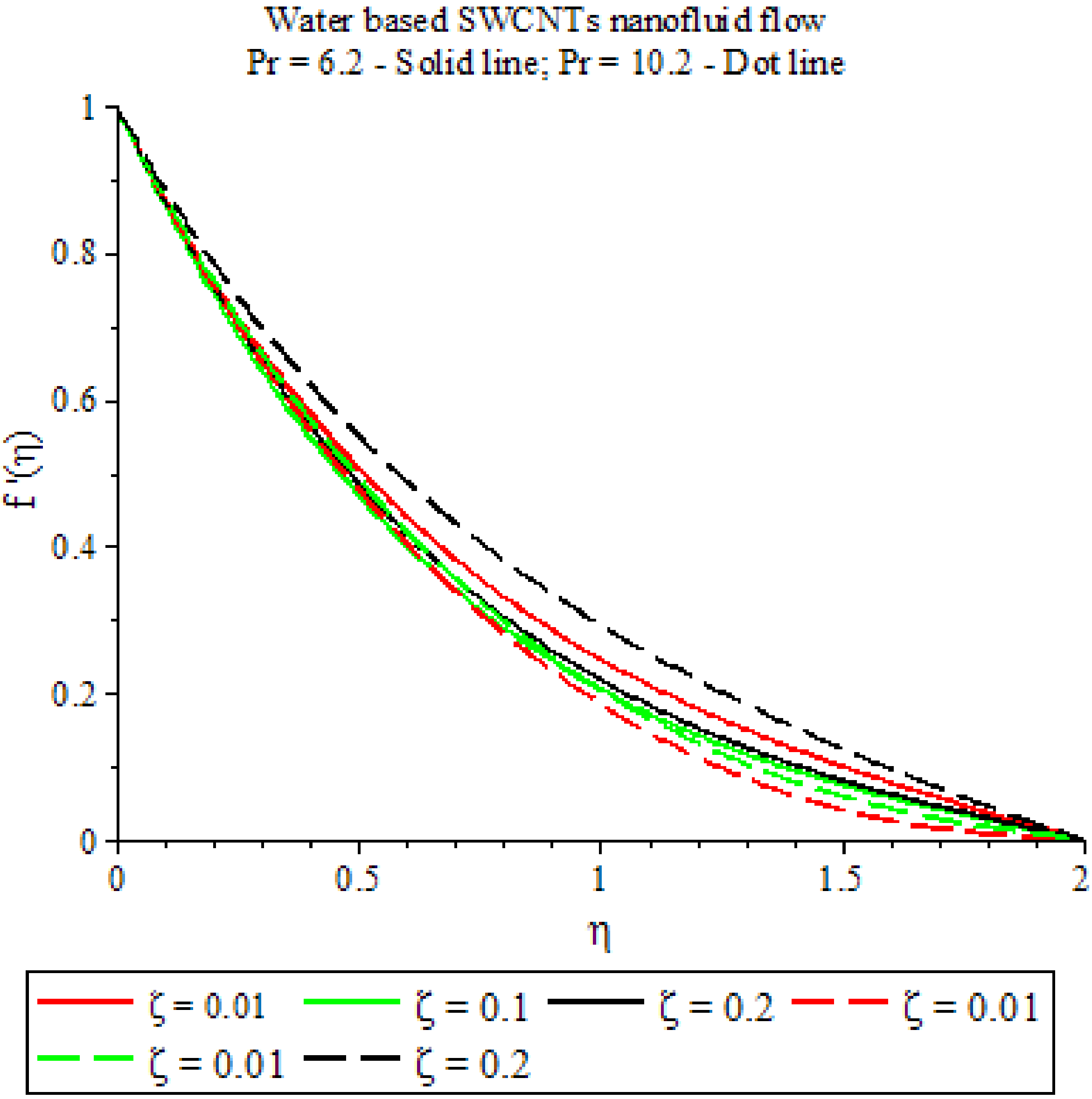


Figure 5.3 Prandtl number effects on velocity profiles

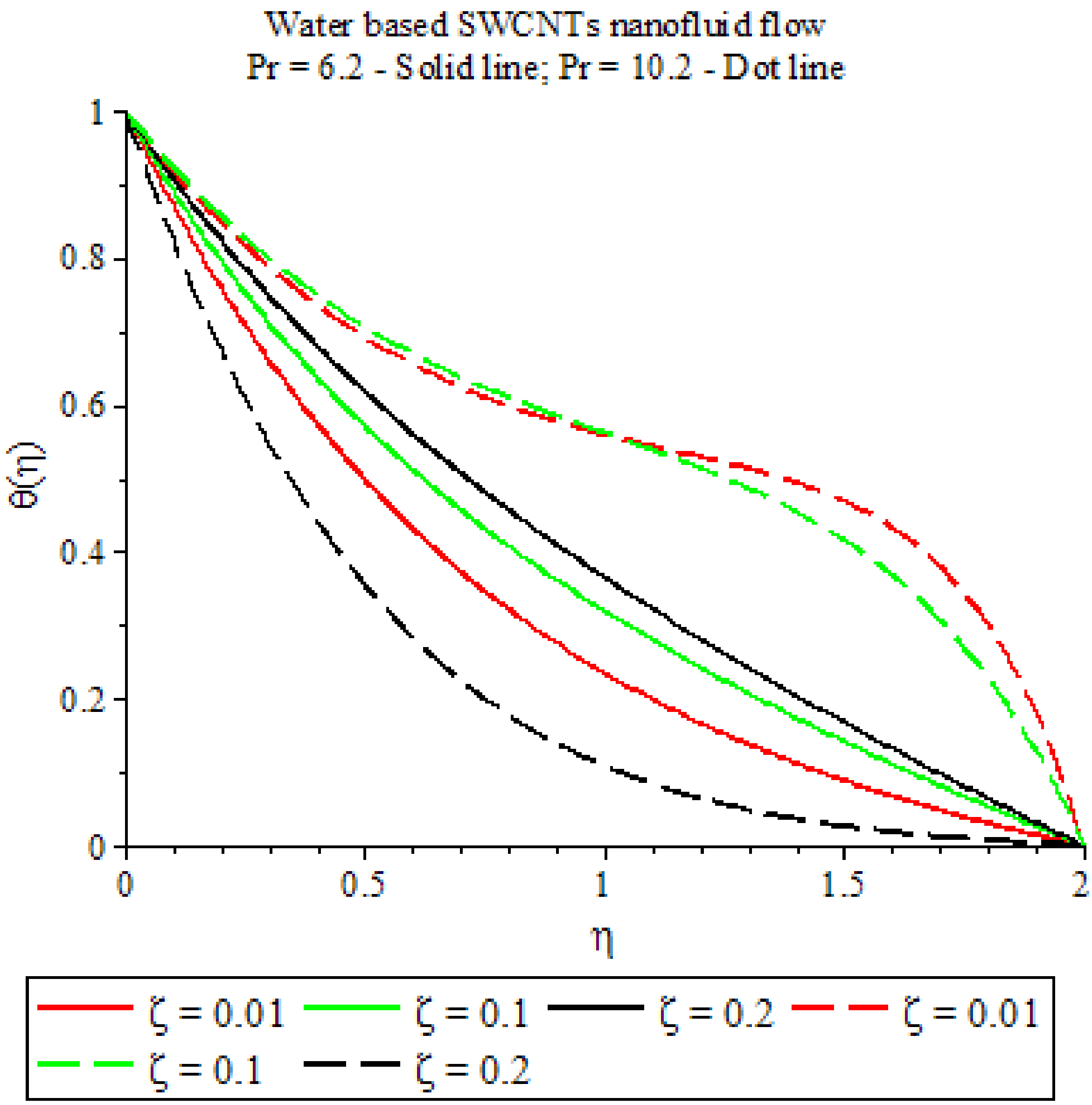


Figure 5.4 Prandtl number effects on temperature profiles

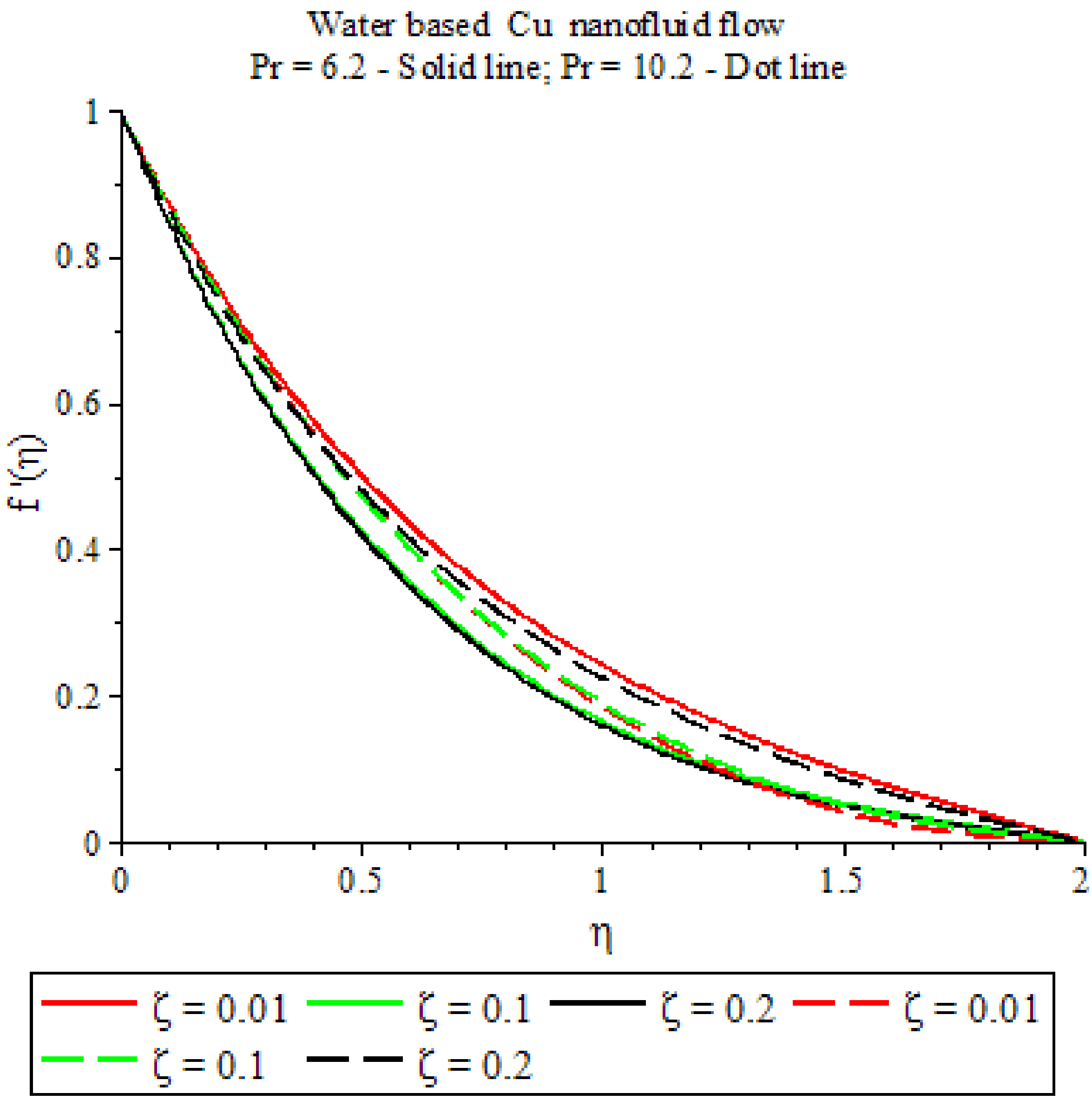


Figure 5.5 Prandtl number effects on velocity profiles

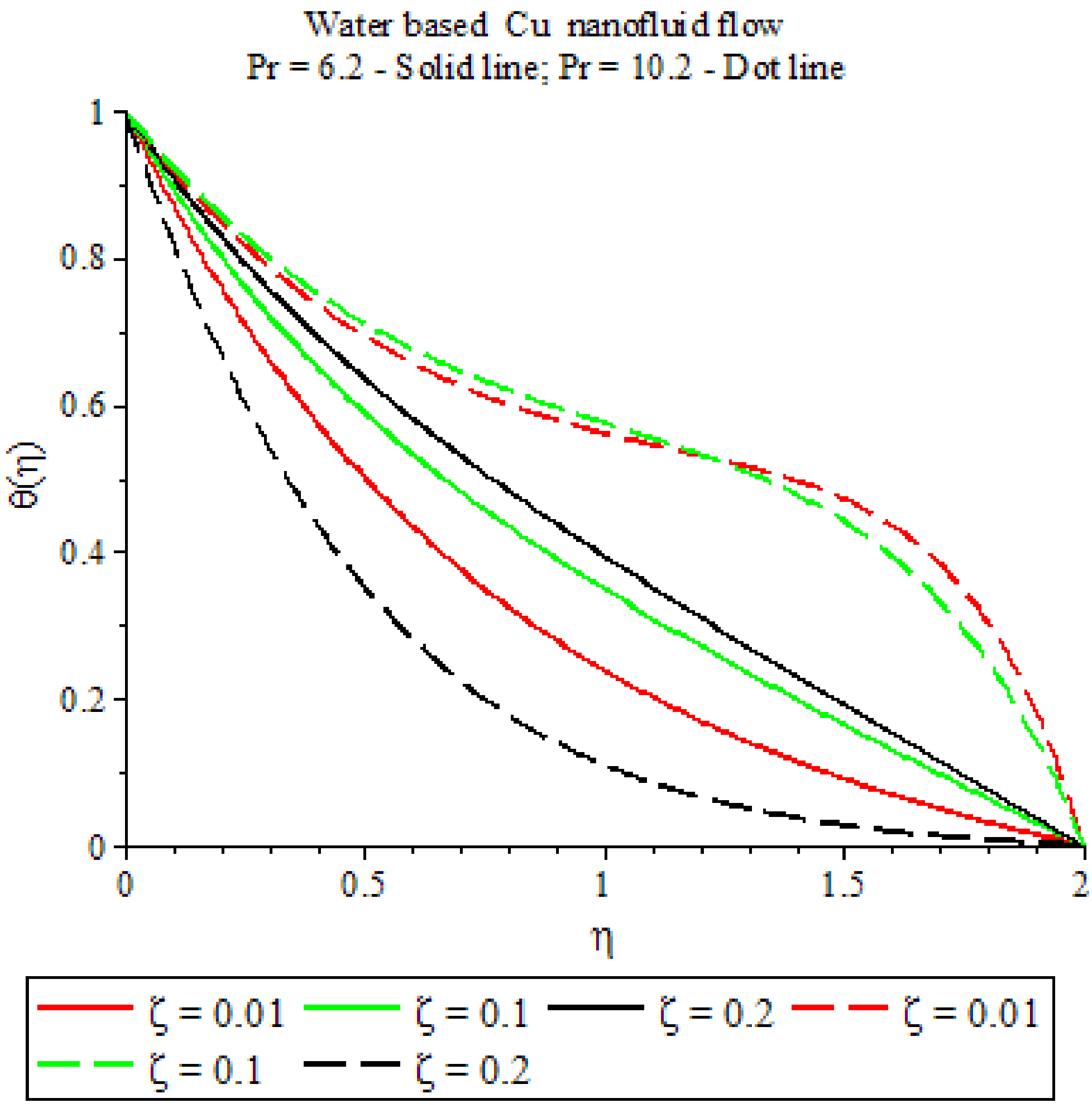


Figure 5.6 Prandtl number effects on temperature profiles

Table 5. 2 Prandtl number on Skin friction and the Rate of heat transfer for the water based SWCNTs and Cu

When $Pr := 6.2;\ S := 0.1; a := 1;\ \delta := 0.1;\ \lambda := 0.1;\ p := 1.667; l := 0.2; m := 3; A := 0.866;$

S.No.1	Parameter	Prandtl number 6.2		Prandtl number 10.2	
	NP Vol. Fra,ζ	$f''(0)$	$-\theta(0)$	$f''(0))$	$-\theta(0)$
SWCNTs – water nanofluid flow					
1	0.01	-1.35137	1.37505	-1.37645	0.87330
2	0.1	-1.47157	1.16633	-1.32887	0.82097
3	0.2	-1.39653	0.98607	-1.21831	1.91076
Cu – water nanofluid flow					
1	0.01	-1.36947	1.37269	-1.38579	0.87325
2	0.1	-1.64938	1.14557	-1.40487	0.82235
3	0.2	-1.66383	0.97215	-1.48634	1.97050

SWCNTs – water nanofluid flow

From the Figs. 5.3 and 5.4 that the velocity of the water based SWCNTs nanofluid flow decreases for Pr = 6.2 and increases for Pr = 10.2 with increase of nanoparticle volume fraction. The temperature of the water based SWCNTs nanofluid flow increases for Pr = 6.2 whereas the temperature firstly decreases and then increases for Pr = 10.2 with increase of nanoparticle volume fraction. The rate of heat transfer for Pr = 6.2 decreases whereas the rate of heat transfer for Pr = 10.2 firstly decreases and then increases with increase of nanoparticle volume fraction.

Cu – water nanofluid flow

From the Figs. 5.5 and 5.6 that the velocity of the water based Cu nanofluid flow decreases for Pr = 6.2 and increases for Pr = 10.2 with increase of nanoparticle volume fraction. The temperature of the water based Cu nanofluid flow increases for Pr = 6.2 whereas the temperature firstly decreases and then increases for Pr = 10.2 with increase of nanoparticle volume fraction. The rate of heat transfer for Pr = 6.2 decreases whereas the rate of heat transfer for Pr = 10.2 firstly decreases and then increases with increase of nanoparticle volume fraction.

Hence, in the presence of nanoparticle volume fraction, the thermal boundary layer thickness of the water based Cu nanofluid flow is stronger as compared to the water based SWCNTs nanofluid flow with increase of Prandtl number.

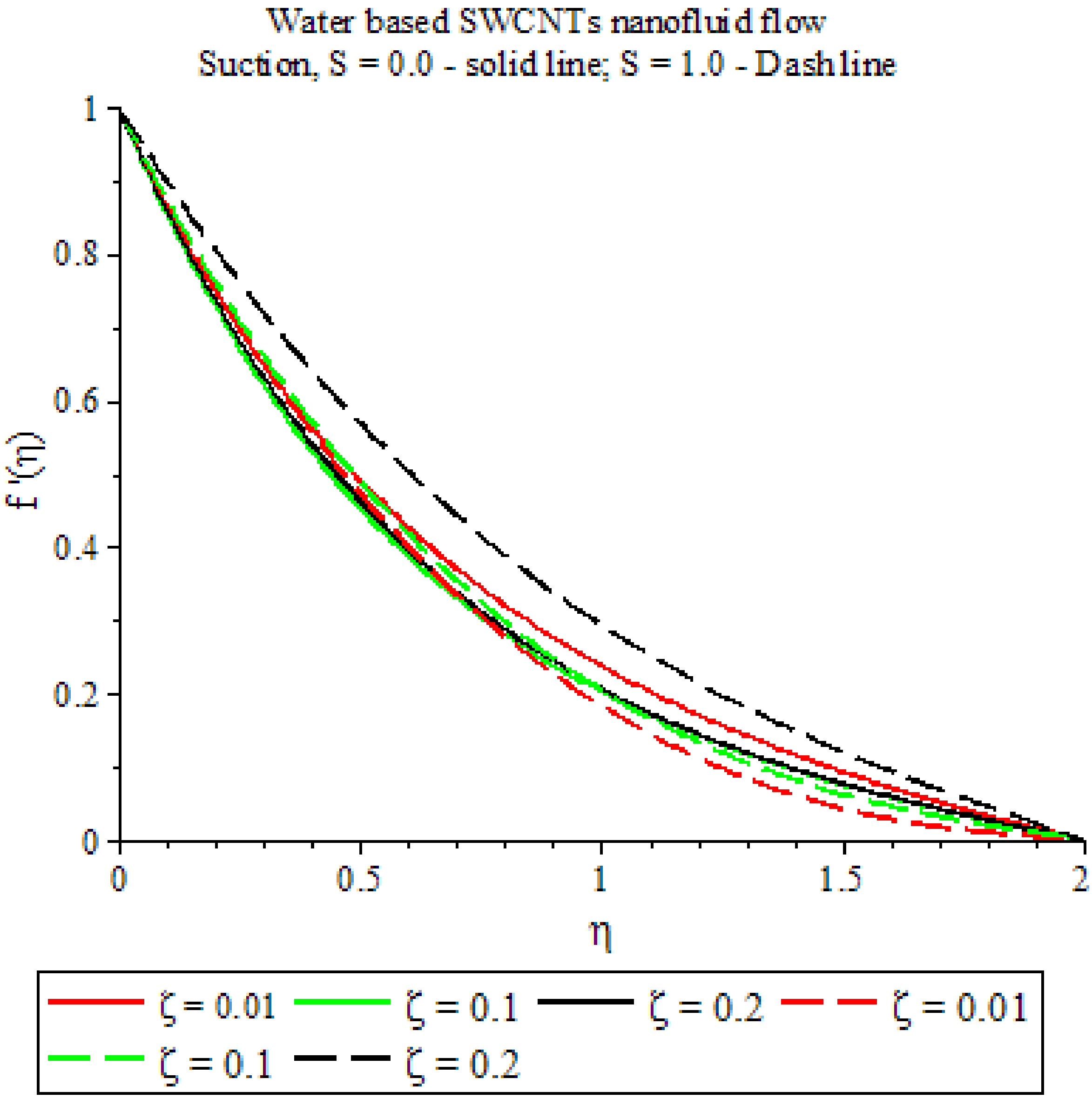


Figure 5.7: Suction effects on velocity profiles

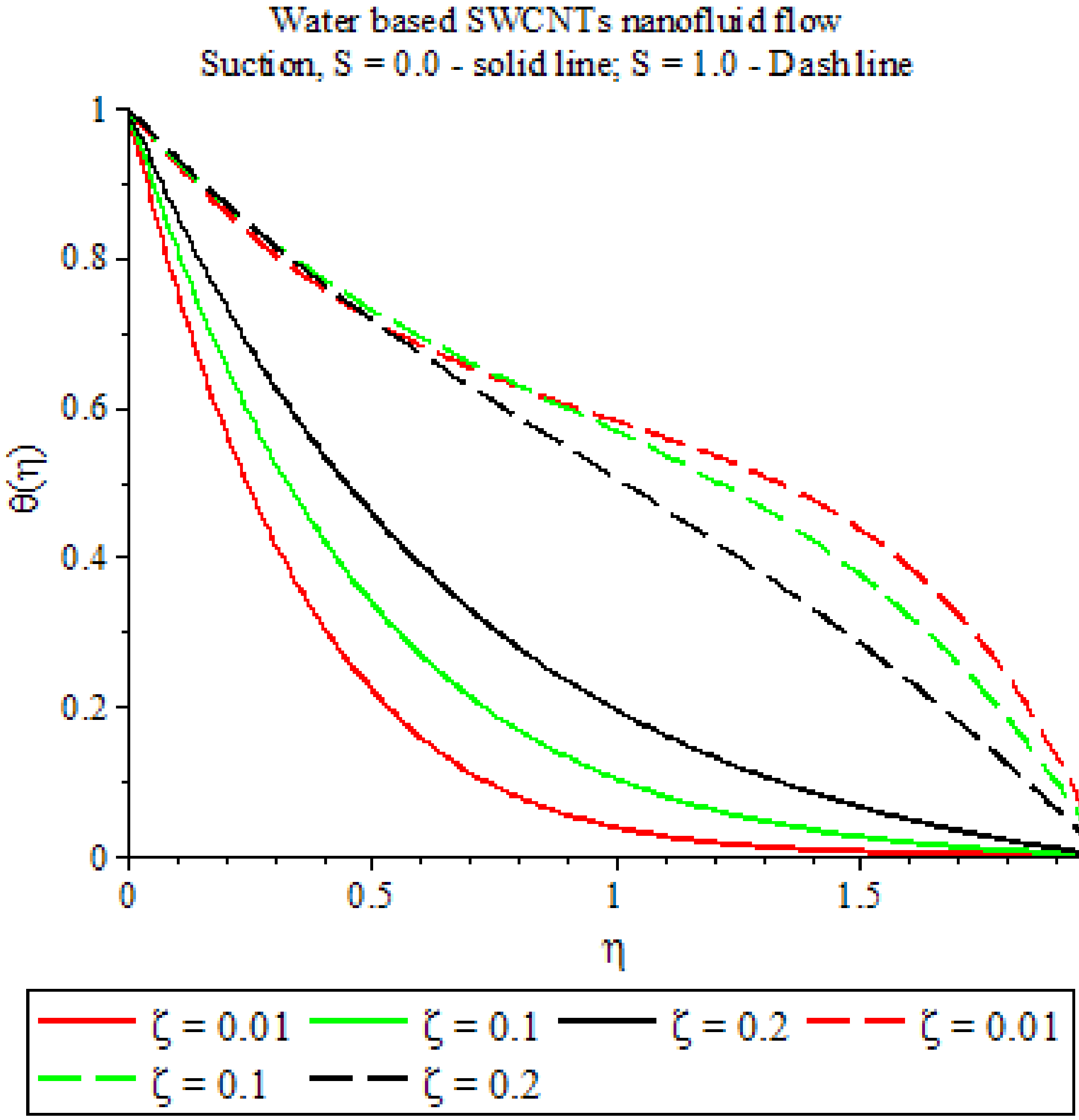


Figure 5.8 Suction effects on temperature profiles

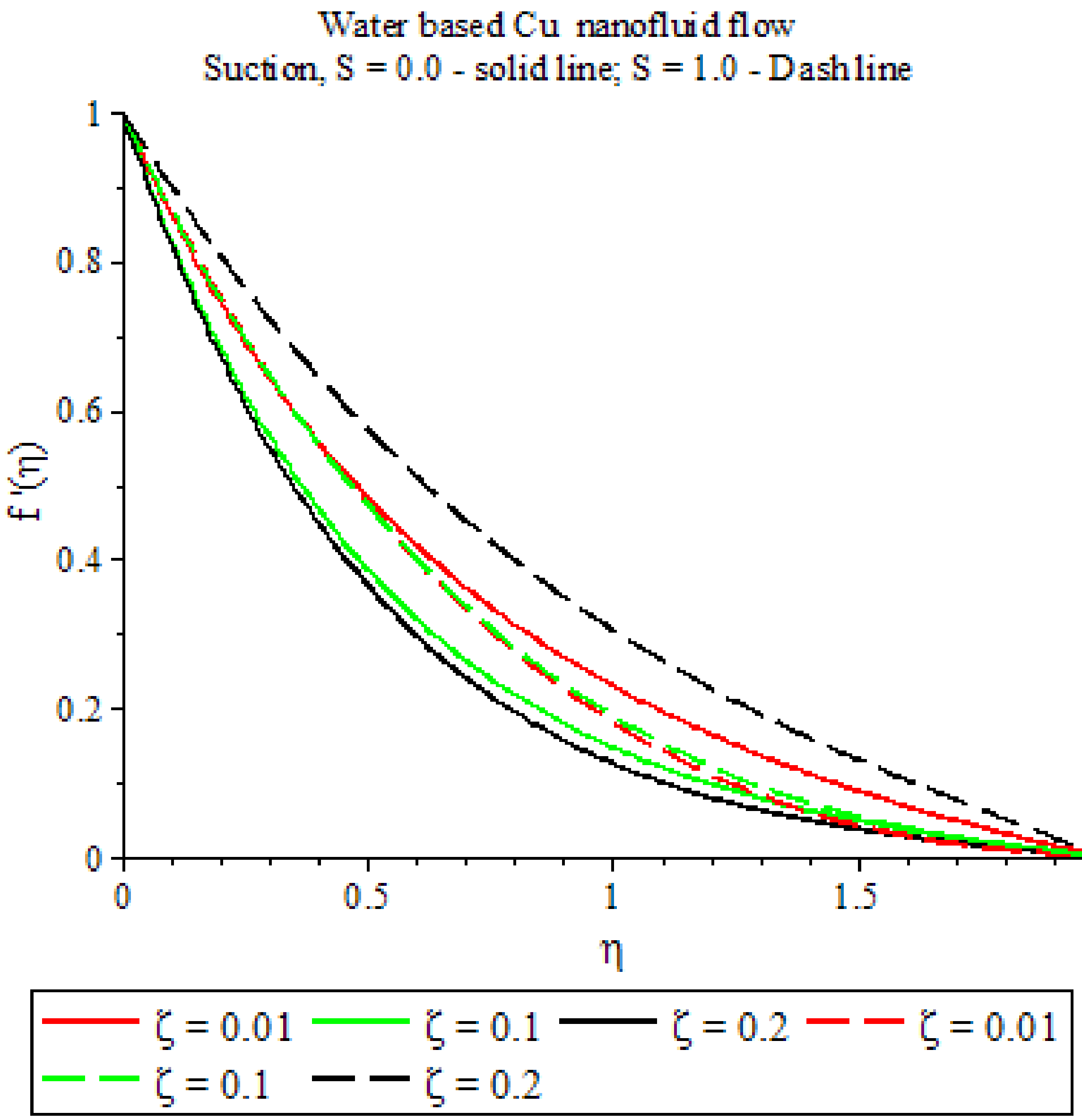


Figure 5.9 Suction effects on velocity profiles

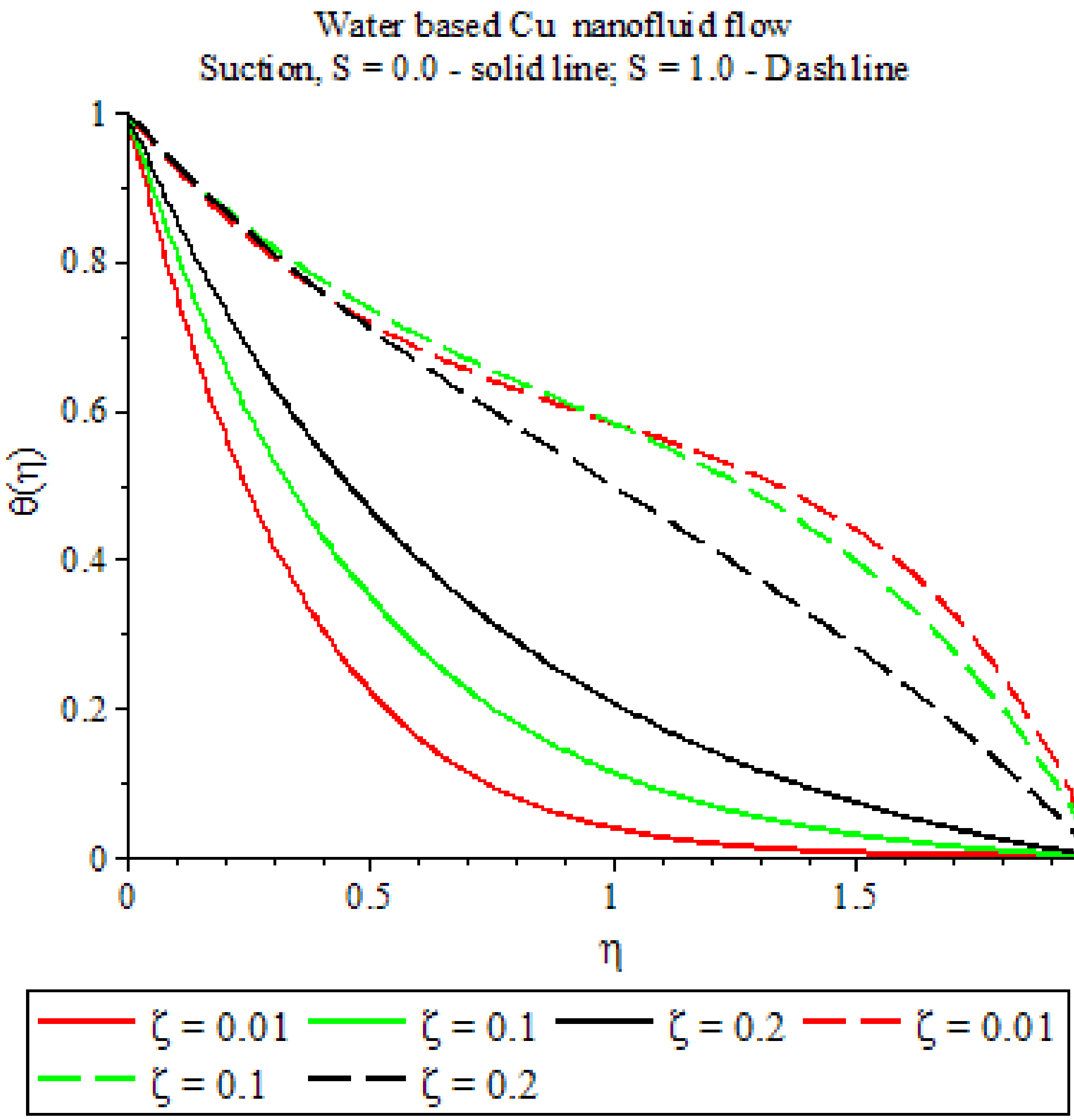


Figure 5.10: Suction effects on temperature profiles

Table 5. 3 Suction on Skin friction and the Rate of heat transfer for the water based SWCNTs and Cu

when $Pr := 6.2;\ S := 0.1; a := 1;\ \delta := 0.1;\ \lambda := 0.1;\ p := 1.667; l := 0.2; m := 3; A := 0.866;$

S.No.1	Parameter	Suction, S = 0.0		Suction, S = 1.0	
	NP Vol. Fra,ζ	$f''(0)$	$-\theta(0)$	$f''(0)$	$-\theta(0)$
SWCNTs – water nanofluid flow					
1	0.01	-1.47597	2.69181	-1.38296	0.80433
2	0.1	-1.60467	2.06152	-1.33372	0.73877
3	0.2	-1.52964	1.53033	-1.06479	0.70057
Cu – water nanofluid flow					
1	0.01	-1.50974	2.69196	-1.39210	0.80432
2	0.1	-1.90631	2.05886	1.40906	0.73953
3	0.2	-1.97734	1.54606	-1.05793	0.72308

SWCNTs – water nanofluid flow

From the Figs. 5.7and 5.8 that the velocity of the water based SWCNTs nanofluid flow decreases for suction, S = 0.0 and increases for S = 1.0 with increase of nanoparticle volume fraction. The temperature of the water based SWCNTs nanofluid flow increases for S = 0.0 whereas the temperature decreases for S = 1.0 with increase of nanoparticle volume fraction. The rate of heat transfer for S = 0.0 firstly decreases and then increases whereas the rate of heat transfer for S = 1.0 decreases with increase of nanoparticle volume fraction.

Cu – water nanofluid flow

From the Figs. 5.9and 5.10 that the velocity of the water based Cu nanofluid flow decreases for suction, S = 0.0 and increases for S = -1.0 with increase of nanoparticle volume fraction. The temperature of the water based Cu nanofluid flow increases for S = 0.0 whereas the temperature decreases for S = -1.0 with increase of nanoparticle volume fraction. The rate of heat transfer for S = 0.0 and S = -1.0 decreases with increase of nanoparticle volume fraction.

Hence, in the presence of nanoparticle volume fraction, the thermal boundary layer thickness of the water based Cu nanofluid flow is stronger as compared to the water based SWCNTs nanofluid flow with increase of Suction parameter.

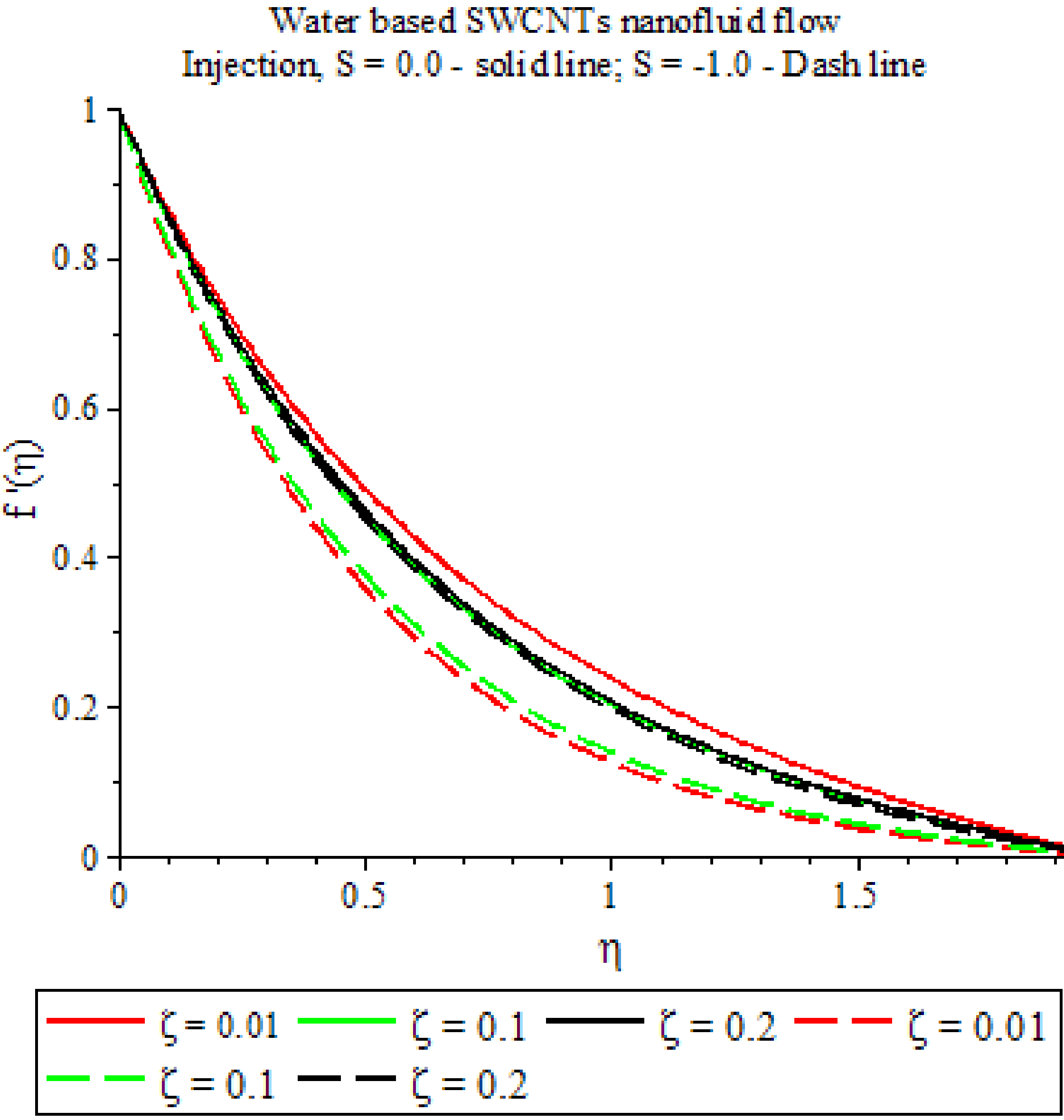


Figure 5.11: Injection effects on velocity profiles

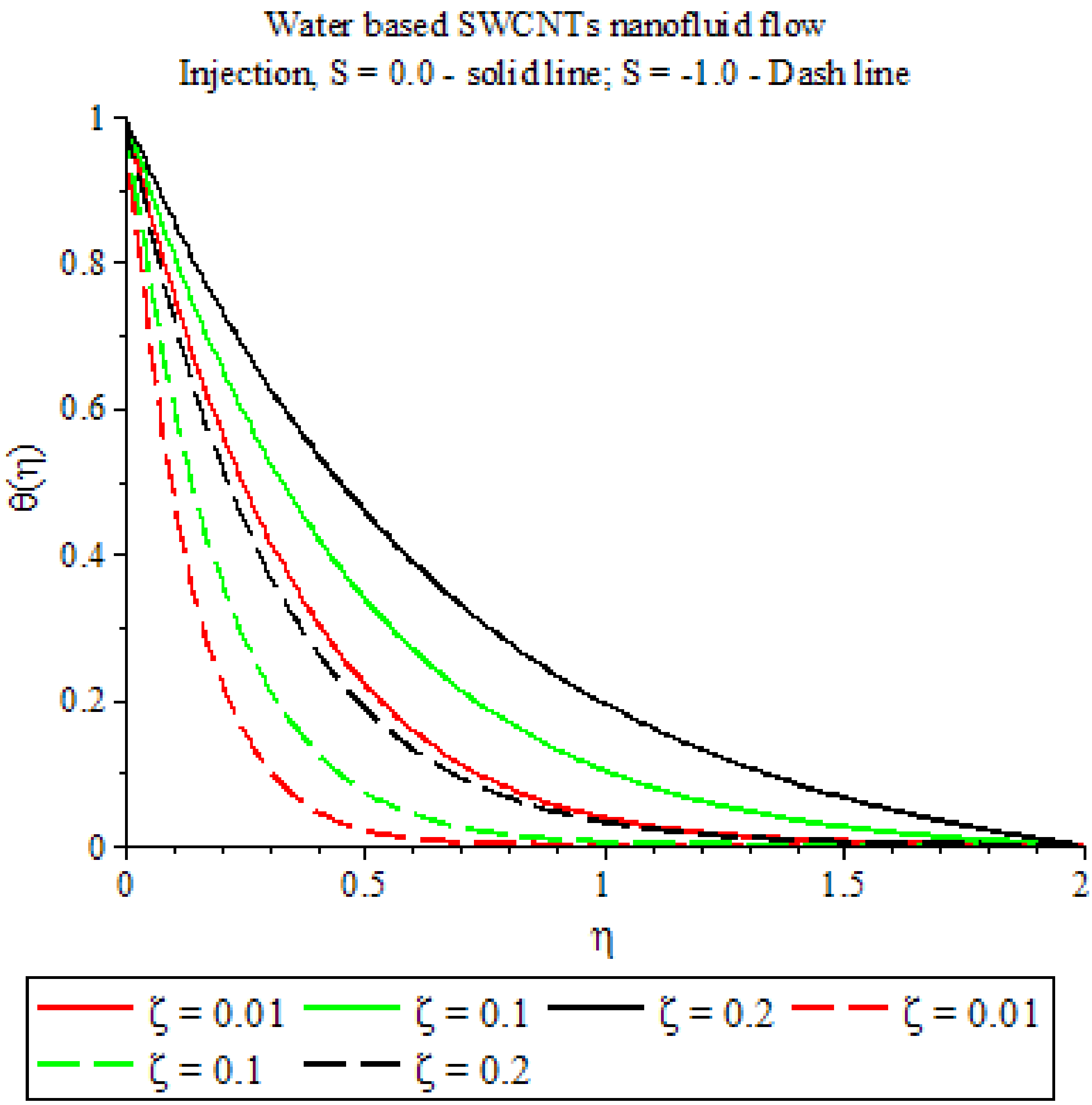


Figure 5.12: Injection effects on temperature profiles

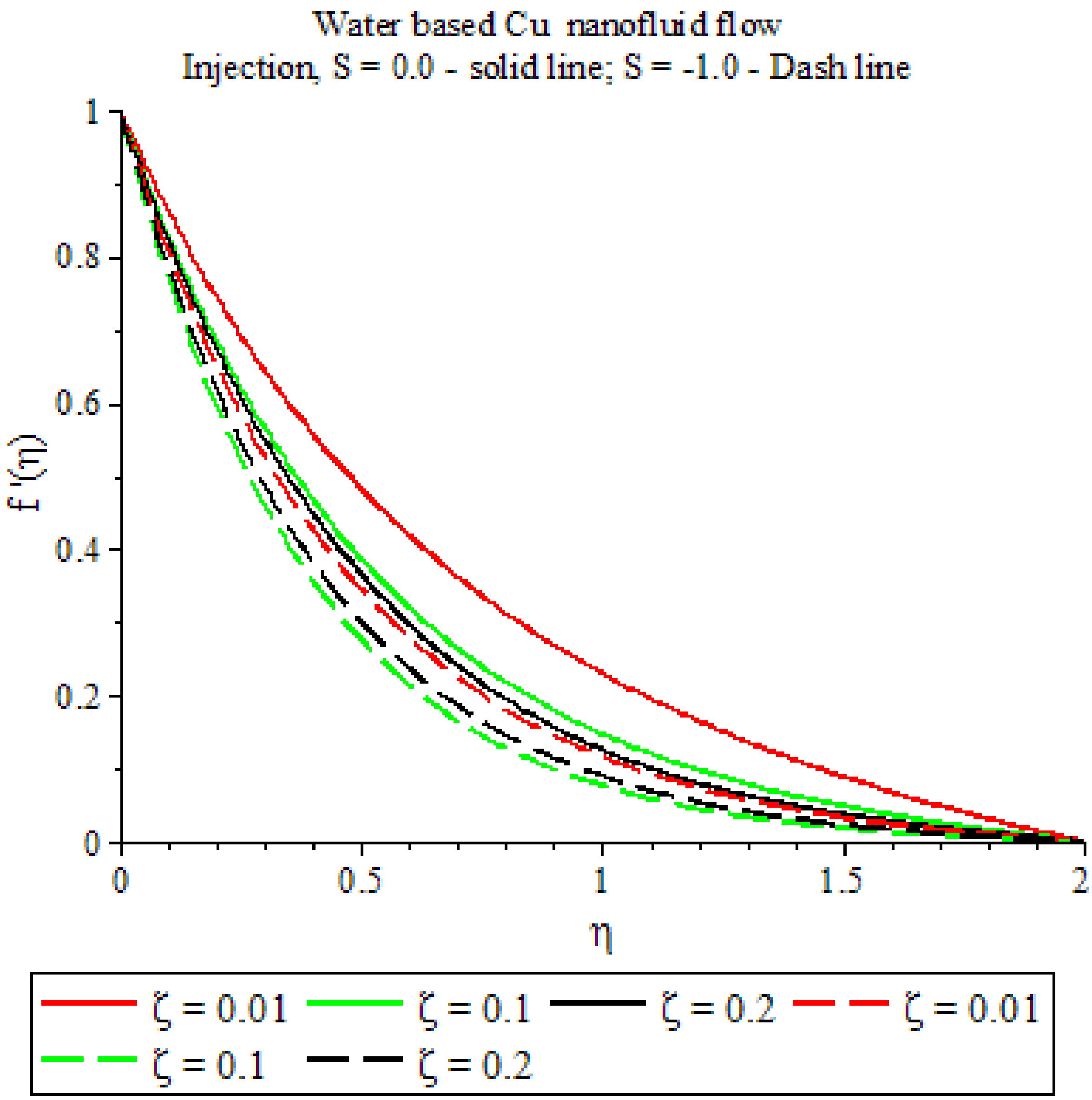


Figure 5.13: Injection effects on velocity profiles

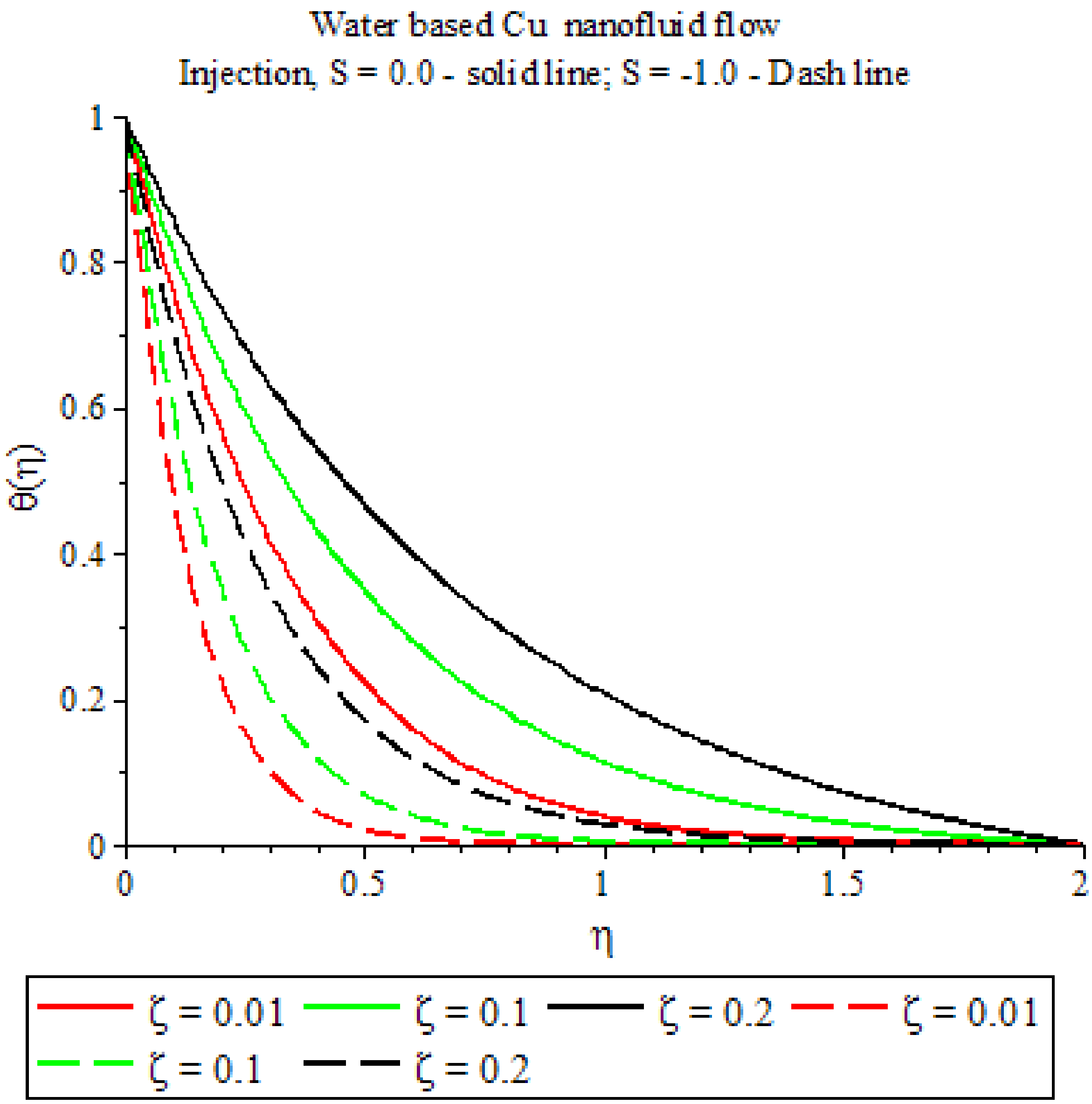


Figure 5.14: Injection effects on temperature profiles

Table 5.4: Injection on Skin friction and the Rate of heat transfer for the water based SWCNTs and Cu

When $Pr := 6.2;\ S := 0.1;\ a := 1;\ \delta := 0.1;\ \lambda := 0.1;\ p := 1.667;\ l := 0.2;\ m := 3;\ A := 0.866;$

S.No.1	Parameter NP Vol. Fra, ζ	Injection, S = 0.0 $f''(0)$	$-\theta(0)$	Injection, S =- 1.0 $f''(0)$	$-\theta(0)$
SWCNTs – water nanofluid flow					
1	0.01	-1.47597	2.69181	-2.11424	7.15812
2	0.1	-1.60467	2.06152	-2.01398	4.91462
3	0.2	-1.52964	1.53033	-1.63584	3.19311
Cu – water nanofluid flow					
1	0.01	-1.50974	2.69196	-2.19435	7.18655
2	0.1	-1.90631	2.05886	-2.63359	5.10811
3	0.2	-1.97734	1.54606	-2.44039	3.44819

SWCNTs – water nanofluid flow

From the Figs. 5.11 and 5.12 that the velocity of the water based SWCNTs nanofluid flow decreases for injection, S = 0.0 and increases for S = -0.1 with increase of nanoparticle volume fraction. The temperature of the water based SWCNTs nanofluid flow increases for S = 0.0 for S = -0.1 with increase of nanoparticle volume fraction. The rate of heat transfer for S = 0.0 and S = -0.1 decreases with increase of nanoparticle volume fraction.

Cu – water nanofluid flow

From the Figs. 5.13 and 5.14 that the velocity of the water based Cu nanofluid flow decreases for injection, S = 0.0 and S = -0.1 with increase of nanoparticle volume fraction. The temperature of the water based Cu nanofluid flow increases for S = 0.0 and S = -0.1 with increase of nanoparticle volume fraction. The rate of heat transfer for injection, S = 0.0 and S = -0.1 increases with increase of nanoparticle volume fraction.

Hence, in the presence of nanoparticle volume fraction, the thermal boundary layer thickness of the water based SWCNTs nanofluid flow is more significant as compared to the water based Cu nanofluid flow with increase of injection parameter.

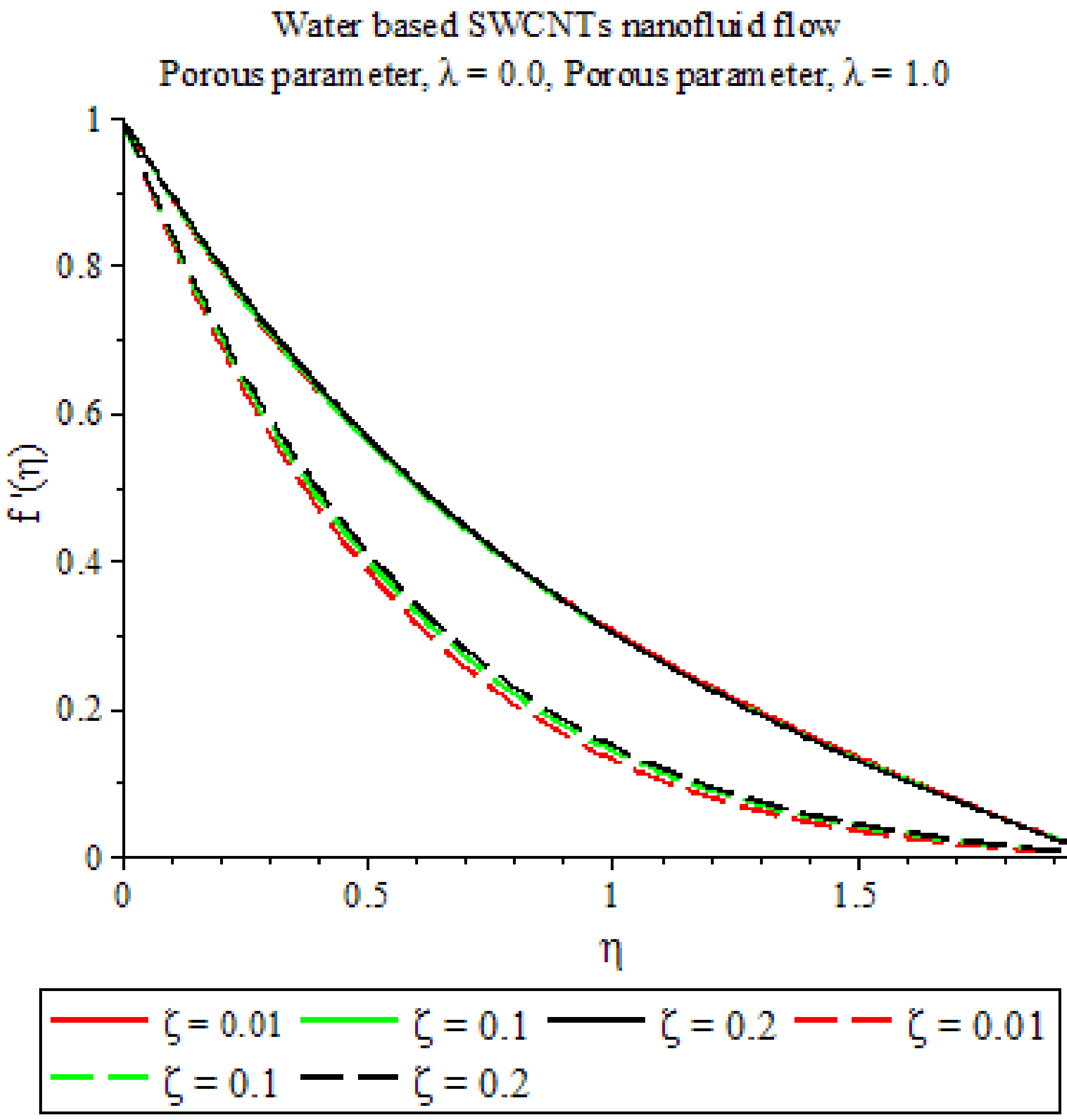


Figure 5.15: Porous effects on velocity profiles

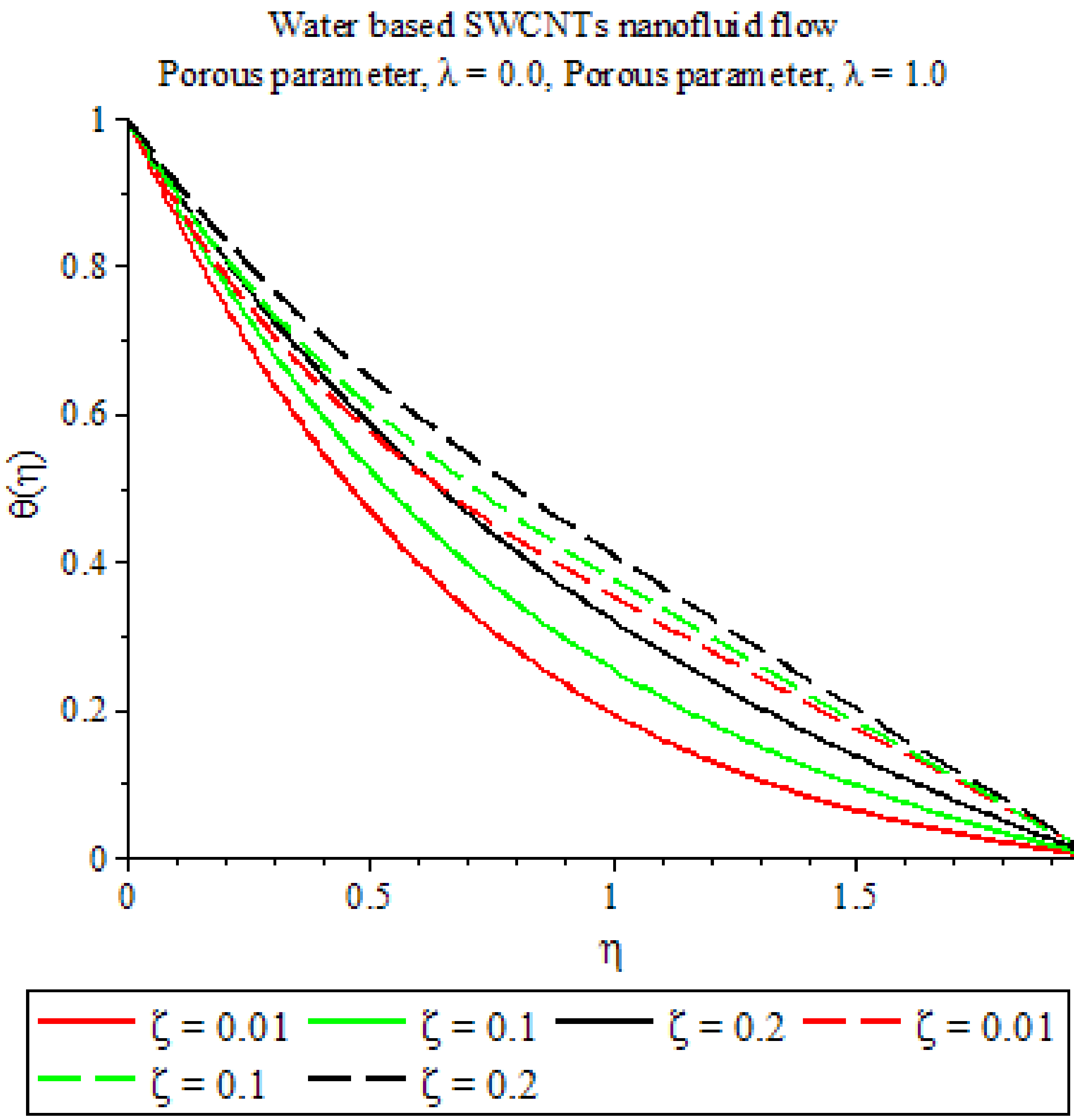


Figure 5.16 Porous effects on temperature profiles

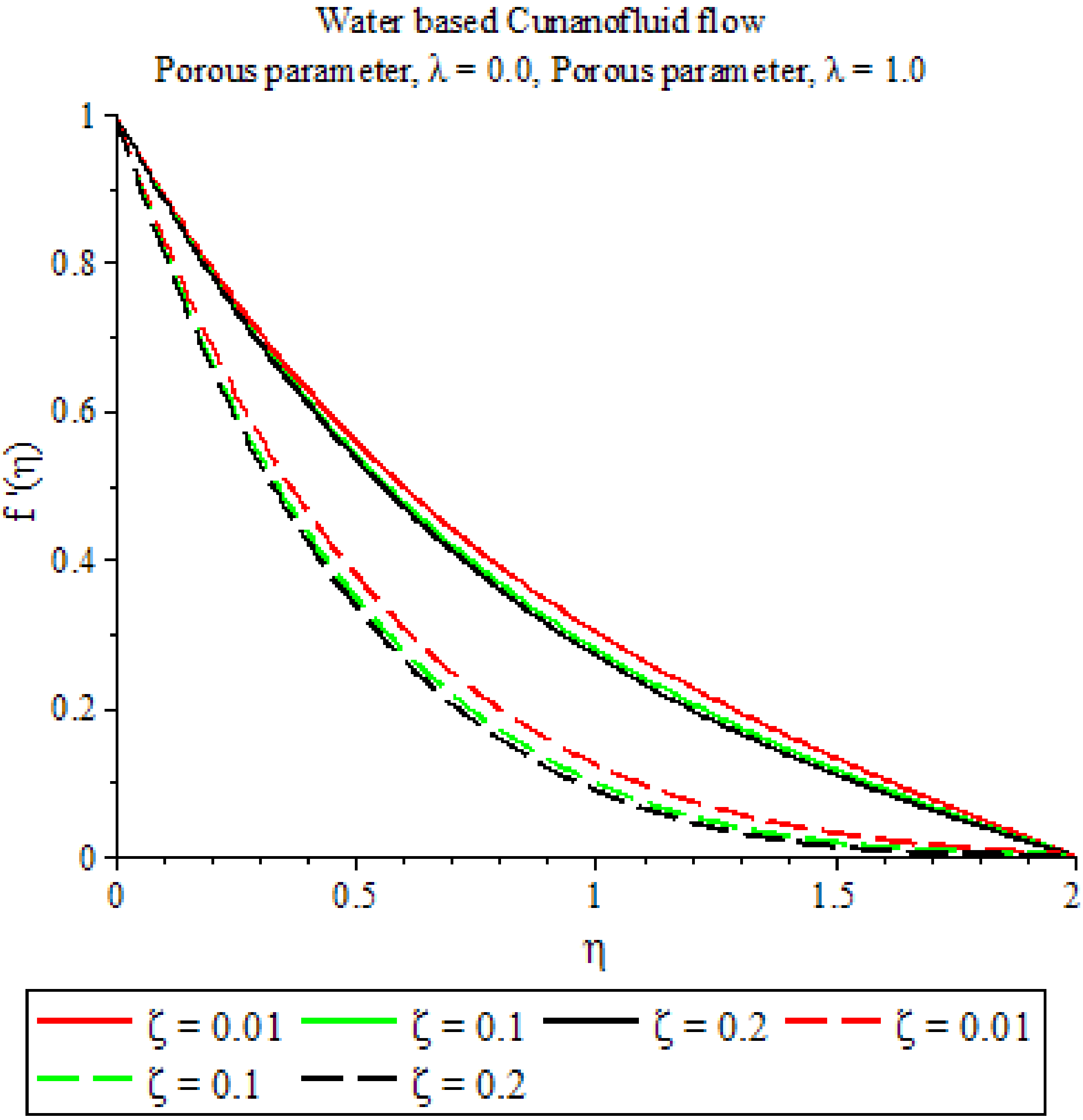


Figure 5.17 porous effects on velocity profiles

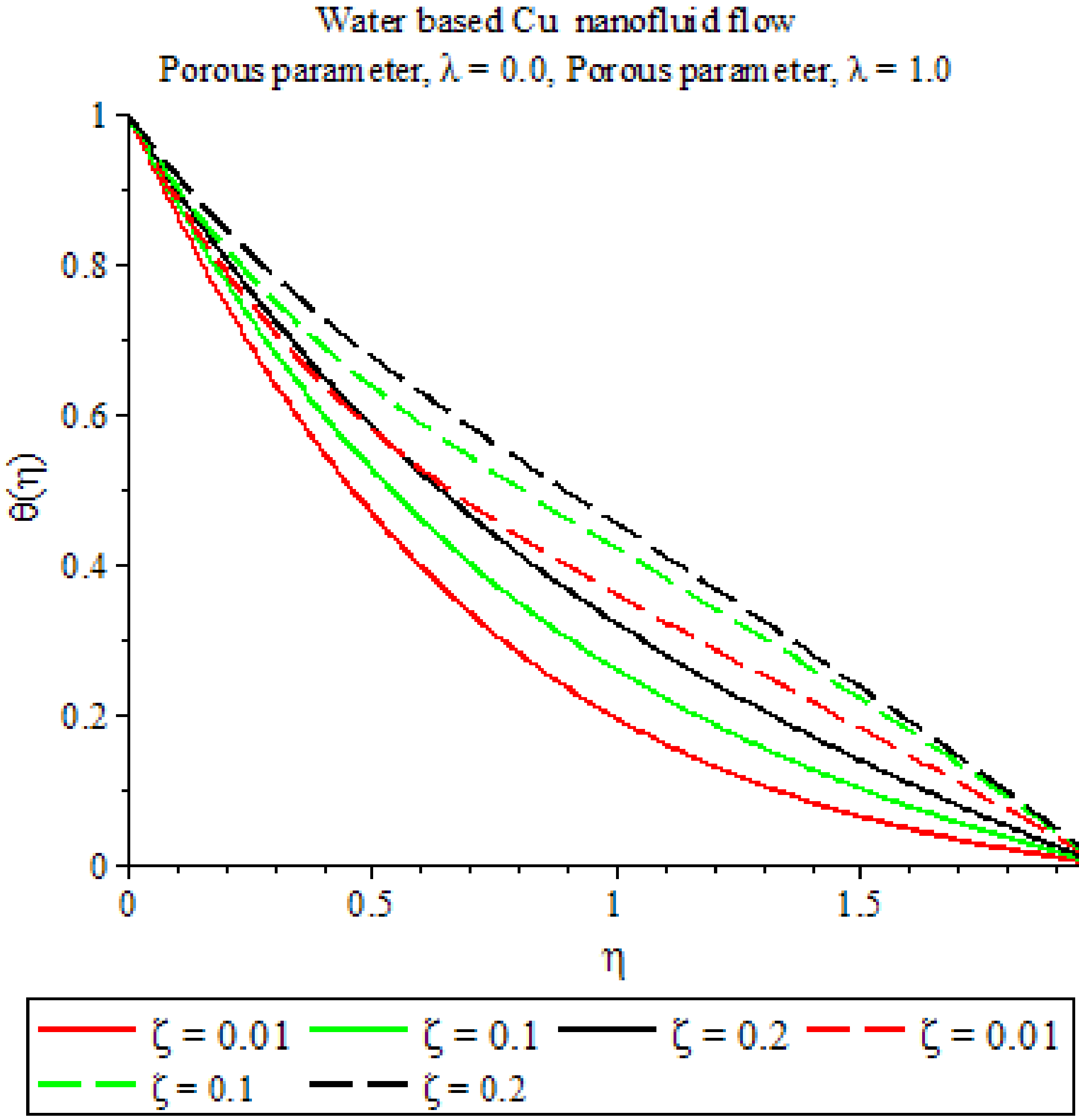


Figure 5.18 porous effects on temperature profiles

Table 5. 5 Porous on Skin friction and the Rate of heat transfer for the water based SWCNTs and Cu

When $Pr := 6.2;\ S := 0.1;\ a := 1;\ \delta := 0.1;\ \lambda := 0.1;\ p := 1.667;\ l := 0.2;\ m := 3;\ A := 0.866;$

S.No.1	Parameter	porous parameter, λ = 0.0		porous parameter, λ = 1.0	
	NP Vol. Fra, ζ	$f''(0)$	$-\theta(0)$	$f''(0)$	$-\theta(0)$
SWCNTs – water nanofluid flow					
1	0.01	-1.15947	1.42437	-1.81770	1.25336
2	0.1	-1.13826	1.25196	-1.74509	1.09724
3	0.2	-1.10398	1.05154	-1.67983	0.92350
Cu – water nanofluid flow					
1	0.01	-1.17004	1.42396	-1.85188	1.24706
2	0.1	-1.21471	1.25722	-1.99673	1.06044
3	0.2	-1.21761	1.07350	-2.05079	0.88953

SWCNTs – water nanofluid flow

From the Figs. 5.15 and 5.16 that the velocity of the water based SWCNTs nanofluid flow decreases for porous parameter, λ = 0.0 and increases for λ = 1.0 with increase of nanoparticle volume fraction. The temperature of the water based SWCNTs nanofluid flow increases for λ = 0.0 and λ = 1.0 with increase of nanoparticle volume fraction. The rate of heat transfer for λ = 0.0 and λ = 1.0 decreases with increase of nanoparticle volume fraction.

Cu – water nanofluid flow

From the Figs. 5.17 and 5.18 that the velocity of the water based Cu nanofluid flow decreases for porous parameter, λ = 0.0 and λ = 1.0 with increase of nanoparticle volume fraction. The temperature of the water based Cu nanofluid flow increases for λ = 0.0 and λ = 1.0 with increase of nanoparticle volume fraction. The rate of heat transfer for λ = 0.0 and λ = 1.0 decreases with increase of nanoparticle volume fraction.

Hence, in the presence of nanoparticle volume fraction, the thermal boundary layer thickness of the water based Cu nanofluid flow is more efficient as compared to the water based SWCNTs nanofluid flow with increase of porous parameter.

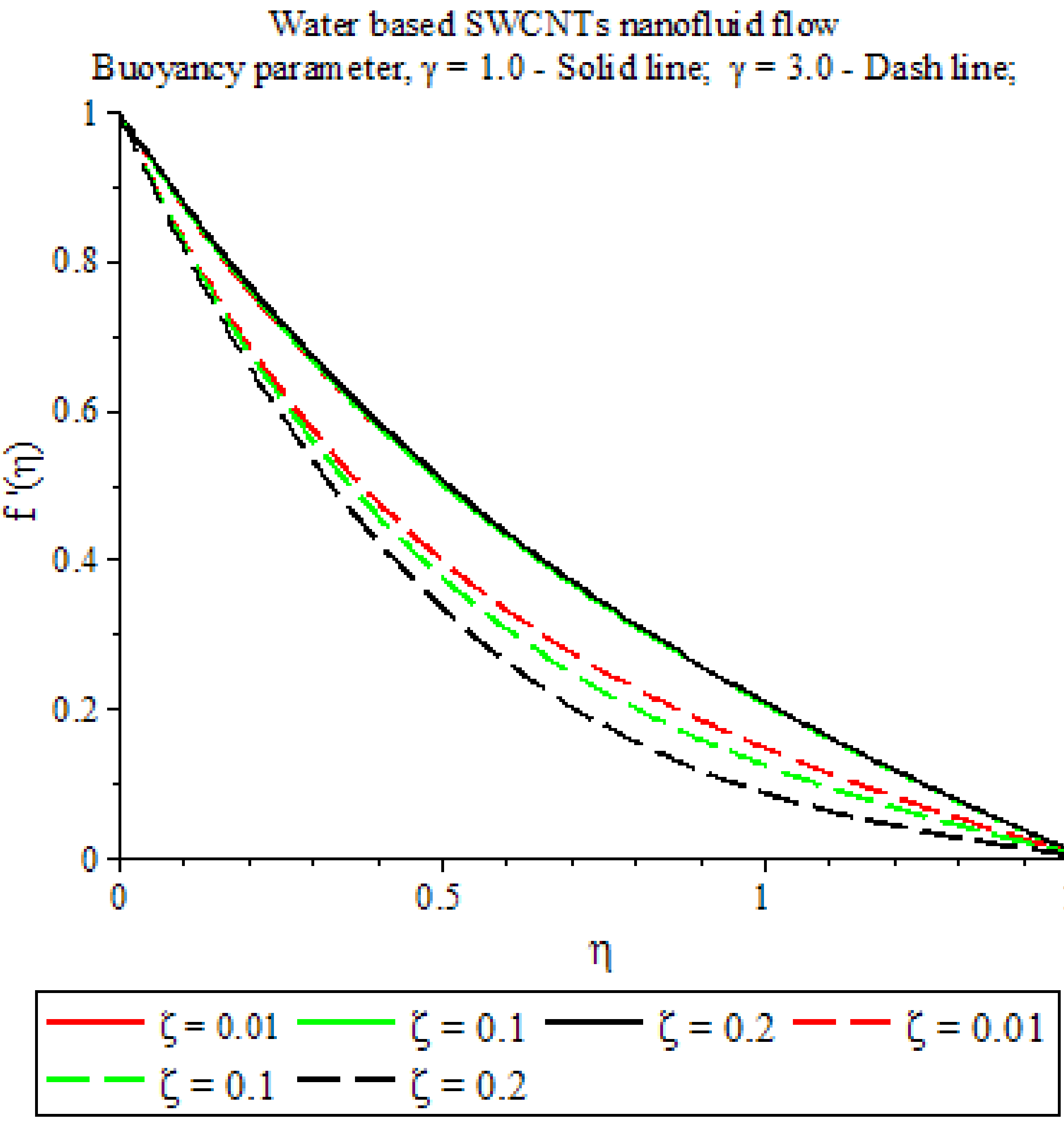


Figure 5.19: Buoyancy effects on velocity profiles

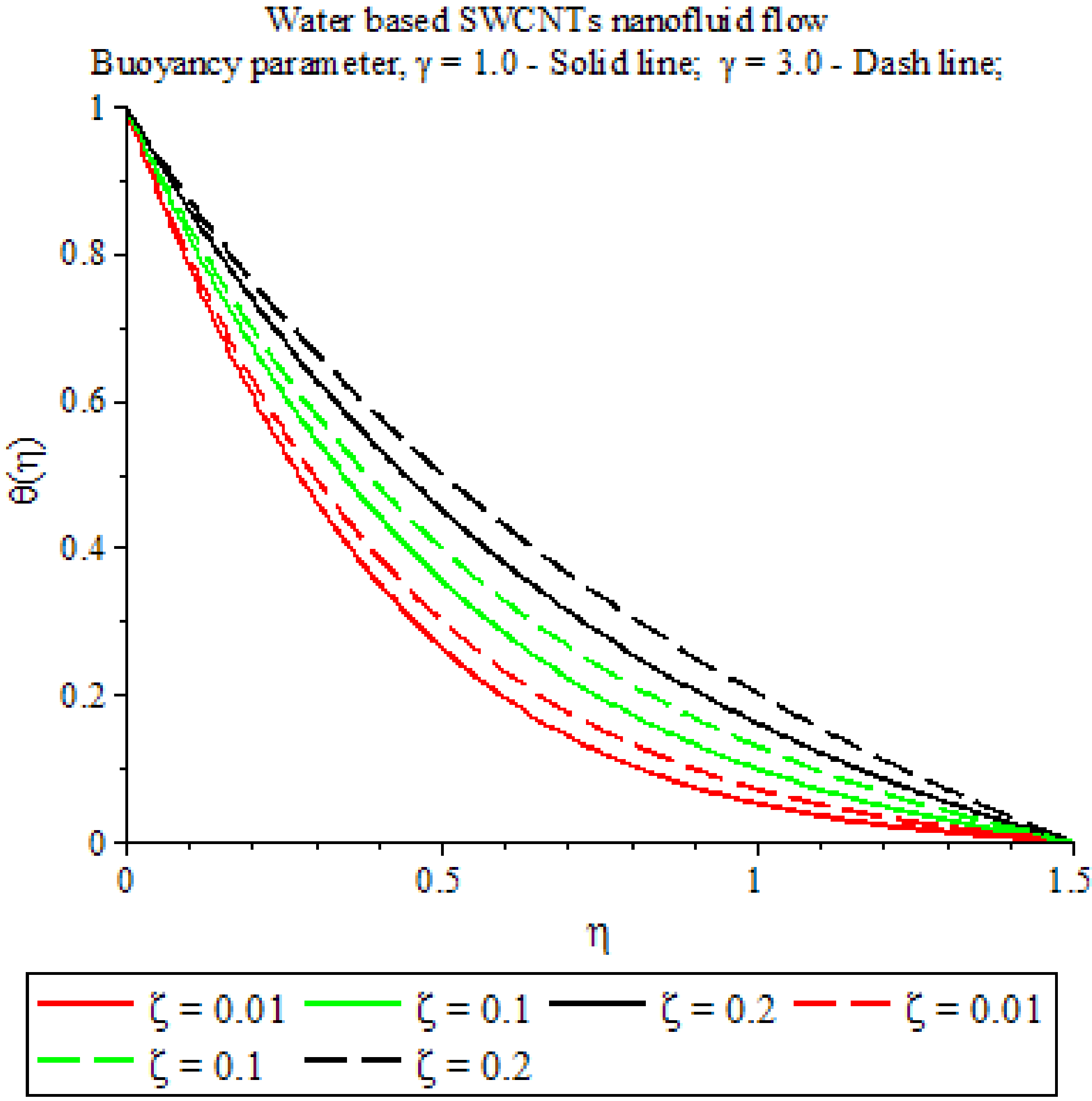


Figure 5.20: Buoyancy effects on temperature profiles

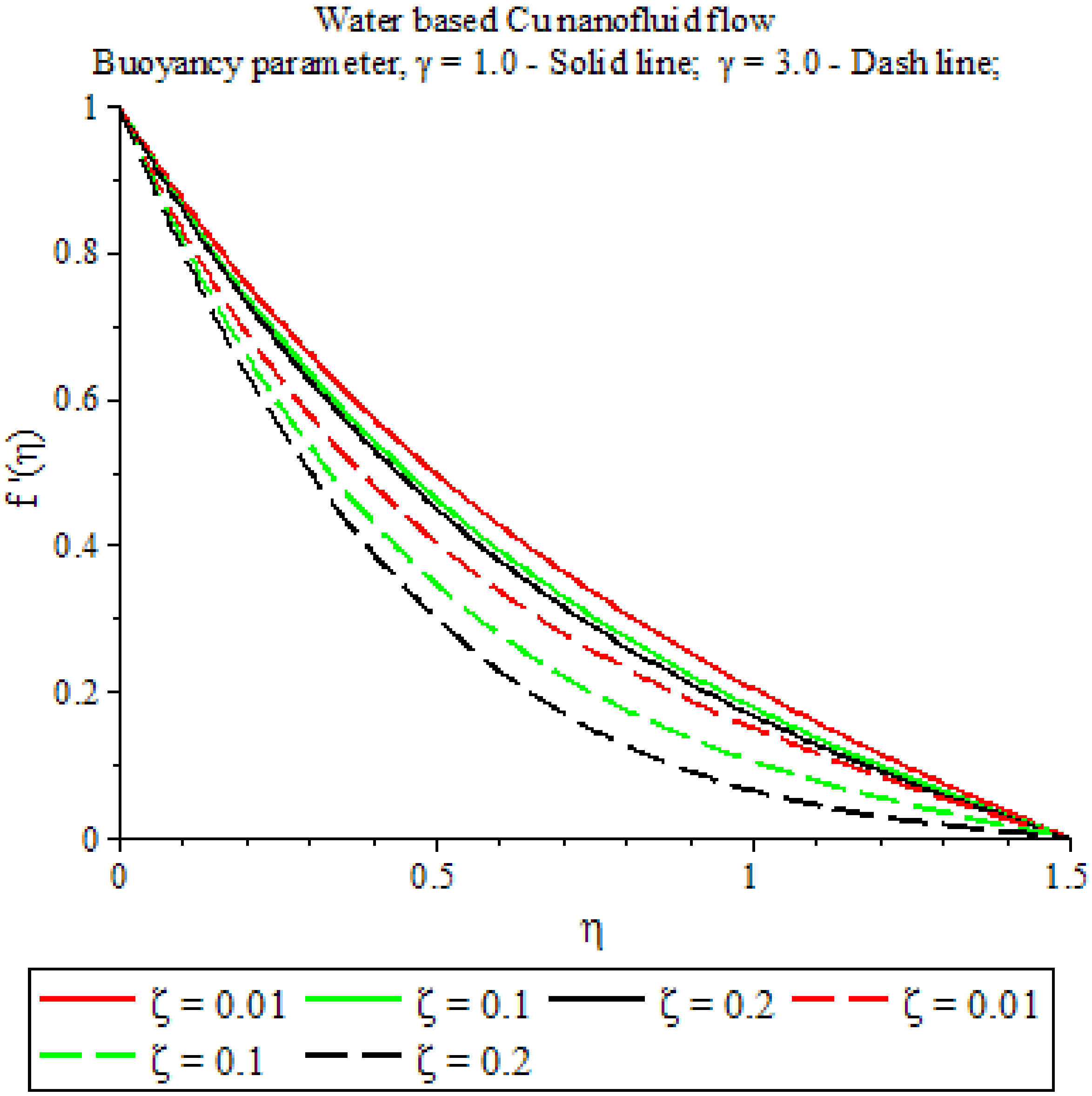


Figure 5.21 Buoyancy effects on velocity profiles

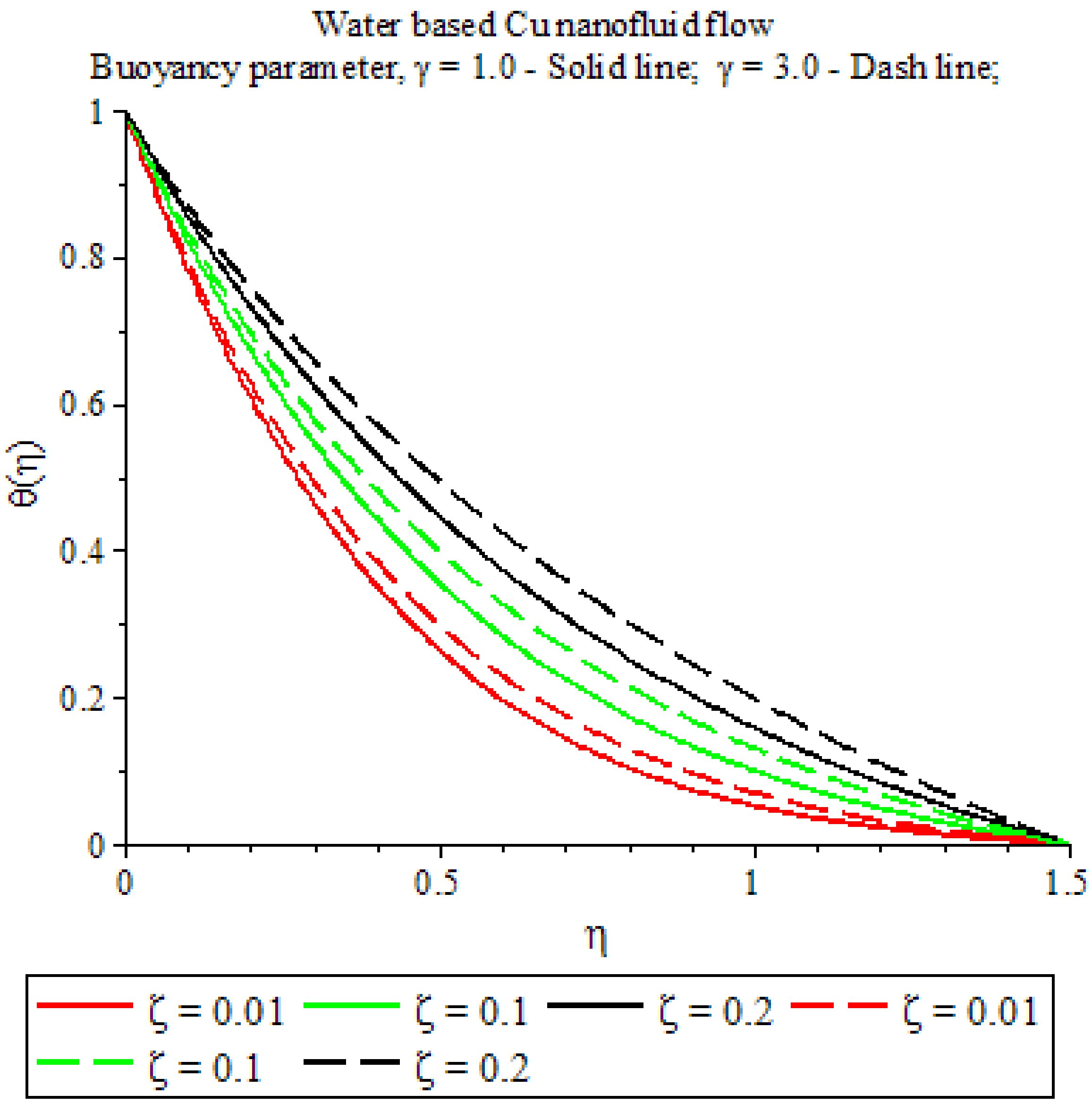


Figure 5.22 Buoyancy effects on temperature profiles

Table 5. 6 Buoyancy effects on Skin friction and the Rate of heat transfer for the water based SWCNTs and Cu

when $Pr := 6.2;\ S := 0.1; a := 1;\ \delta := 0.1;\ \lambda := 0.1;\ p := 1.667; l := 0.2; m := 3; A := 0.866;$

S.No.1	Parameter NP Vol. Fra, ζ	Buo.Parameter, γ = 1.0 $f''(0)$	$-\theta(0)$	Buo. Parameter, γ = 3.0 $f''(0)$	$-\theta(0)$
SWCNTs – water nanofluid flow					
1	0.01	-1.35833	2.37846	-1.89327	2.25586
2	0.1	-1.33623	1.91386	-1.90969	1.78702
3	0.2	-1.29244	1.48802	-1.99737	1.35692
Cu – water nanofluid flow					
1	0.01	-1.37988	2.37995	-1.87537	2.26772
2	0.1	-1.49871	1.93563	-2.04400	1.81037
3	0.2	-1.53390	1.53507	-2.17712	1.40552

SWCNTs – water nanofluid flow

From the Figs. 5.19 and 5.20 that the velocity of the water based SWCNTs nanofluid flow uniform for buoyancy parameter, γ= 1.0 and decreases for γ = 3.0 with increase of nanoparticle volume fraction. The temperature of the water based SWCNTs nanofluid flow increases for γ = 1.0 and γ = 3.0 with increase of nanoparticle volume fraction. The rate of heat transfer for γ = 1.0 and γ = 3.0 decreases with increase of nanoparticle volume fraction.

Cu – water nanofluid flow

From the Figs. 5.21 and 5.22 that the velocity of the water based Cu nanofluid flow decreases for buoyancy parameter, γ = 1.0 and γ = 3.0 with increase of nanoparticle volume fraction. The temperature of the water based Cu nanofluid flow increases for γ = 1.0 and γ = 3.0 with increase of nanoparticle volume fraction. The rate of heat transfer for $\gamma = 1.0$ and $\gamma = 3.0$ decreases with increase of nanoparticle volume fraction.

Hence, in the presence of nanoparticle volume fraction, the thermal boundary layer thickness of the water based Cu nanofluid flow is stronger as correlated to the water based SWCNTs nanofluid flow with increase of buoyancy parameter.

CHAPTER SIX

6 CONCLUSIONS AND RECOMMENDATIONS

6.1 Conclusions

The model used for the water based Cu and SWCNTs nanofluid incorporates the effects of heat transfer in the presence of buoyancy force field. It is found that the volume fraction of nanoparticles is a key parameter for investigating the impact of nanofluid flow fields. The governing partial differential equations are transformed into ordinary differential equations by applying similarity transformation. These equations are solved numerically using Runge Kutta Fehlberg based shooting method. Numerical results for temperature and velocity are presented graphically for various parameter conditions. Based on the obtained graphical results, the following conclusions can be summarized as follows:

In the presence of nanoparticle volume fraction

- Thermal boundary layer thickness of the water based Cu nanofluid flow is stronger as compared to the water based SWCNTs nanofluid flow with increase of Prandtl number.
- Temperature and thermal boundary layer thickness of the water based Cu nanofluid flow is higher as compared to the water based SWCNTs nanofluid flow with increase of suction parameter.
- Thermal boundary layer thickness of the water based SWCNTs nanofluid flow is more significant as correlated to the water based Cu nanofluid flow with increase of injection parameter.
- Temperature and thermal boundary layer thickness of the water based Cu nanofluid flow is more efficient as compared to the water based SWCNTs nanofluid flow with increase of porous parameter.
- Thermal boundary layer thickness of the water based Cu nanofluid flow is stronger as correlated to the water based SWCNTs nanofluid flow with increase of buoyancy parameter.

Nanofluids are vital due to the fact they may be used in sever a programs related to heat switch and other applications which includes in detergency. Specifically, nanoparticles have lately received huge hobby in diverse fields, together with medication and biomedicine. Nanofluid drift in porous media play a very critical position on programs in engineering as put up-accidental warmth removal in nuclear reactors, sun collectors, drying processes and warmth exchangers, and many others. The interdisciplinary nature of nanofluid research offers a first rate opportunity for exploration and discovery at the frontiers of nanotechnology. Therefore, the concern of nanofluids is of extraordinary hobby worldwide for basic and carried out research.

6.2 Recommendations

6.2.1 Theoretical aspects

The work supplied in this thesis provides the theoretical framework for studying more complicated situations. In the destiny, we will increase our works on unsteady of two or 3-dimensional laminar/turbulent boundary layer flow of various fluids (which includes a energy regulation fluids, micro polar fluids, viscoelastic fluids) over shifting surfaces with numerous geometrical in shapes (horizontal, vertical and willing plate, cone, round cylinder, sphere and aerofoil).

The computational technique used to solve the trouble is similarity technique that is solved numerically using the Runge KuttaFehlberg along with shooting technique. For the future look at, it is possible to increase our work to the local non-similarity solution up to third level truncation or the use of different techniques to locate the exact answer to improve the accuracy.

6.2.2 Experimental factors

Mathematical modelling of water based Cu/SWCNTs nanofluid flow, heat transfer over a linearly porous stretching sheet embedded in a porous medium in the presence of suction/injection with a few prescribed parameters outcomes has been investigated. This model offers us the information to understand the behavior of velocity, temperature and rate of heat transfer inside the boundary layer. Based totally on theoretical modelling technique, the conduct of boundary layer can be managed through a few prescribed parameters. But, for experimentalists, this model may be used to guide or justify the experimental take a look at or vice versa.

6.2.3 Engineering factors

Nowadays, the issues of the growing air and water pollution in the industrial nations are critical due to the decreasing first-rate of live. To cast off that pollutant debris from our surroundings, we want a few pollutant elimination equipment's. Consequently, from engineering or manufacturing aspects, the information on specific nanofluids may be used to layout the tool/device for getting rid of pollutant debris from our surroundings.

In the end, it is hoping that the prevailing research may be useful to study the engineering problems which include a migration of moisture in the heat exchanger devices, petroleum reservoirs, filtration, chemical catalytic reactors and strategies, nuclear waste repositories, spreading of chemical pollutants in plants, diffusion of drug treatments in blood veins and extraction of geothermal energy.

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Appendix A

Maple program for nanoparticle volume fraction on water based Cu and SWCNTs nanofluid flow over a vertical stretching surface

$> restart;$

$>Shootlib := \text{"C:\\\\shooting/"} :$

$>libname := Shootlib, libname :$

$>with(Shoot) :$

$>with(plots) :$

$>Pr := 6.2;\ S := 0.1;\ \Upsilon := 0.01; p := 1; \lambda := 0.1;$

$$Pr := 6.2$$

$$S := 0.1$$

$$\Upsilon := 0.01$$

$$p := 1$$

$$\lambda := 0.1$$

$> (\rho s) := 2600;\ (cps) := 425; (ks) := 6600; (\beta s) := 0.99\mathrm{E}{-5}; (\rho f) := 997; (cpf) := 4179;$
$(kf) := 0.613;\ (\beta f) := 63\mathrm{E}{-5};$

$$\rho s := 2600$$

$$cps := 425$$

$$ks := 6600$$

$$\beta s := 0.0000099$$

$$\rho f := 997$$

$$cpf := 4179$$

$$kf := 0.613$$

$$\beta f := 0.00063$$

$>(Kfn) := (ks) + 2.(kf) - 2.\ \Upsilon.((kf) - (ks));$

$$Kfn := 6733.21374$$

$>A := (1 - \Upsilon)^{2.5}.\left(1 - \Upsilon + \Upsilon.\frac{(\rho s)}{(\rho f)}\right);$

$$A := 0.9908664759$$

$>C := \left(1 - \Upsilon + \Upsilon.\frac{(\rho s).(cps)}{(\rho f).(cpf)}\right);$

$$C := 0.9926521296$$

$>B := \left(1 - \Upsilon + \Upsilon.\frac{(\beta s)}{(\beta f)}\right);$

$$B := 0.9901571429$$

$>(Kf) := (ks) + 2.(kf) + 2.\ \Upsilon.((kf) - (ks));$

$$Kf := 6469.23826$$

$>E := \frac{Kfn}{Kf};$

$$E := 1.040804724$$

>

>$FNS := \{f(\eta), u(\eta), v(\eta), \theta(\eta), w(\eta)\}$:

> $ODE := \Big\{\mathrm{diff}(f(\eta),\eta) = u(\eta), \mathrm{diff}(u(\eta),\eta) = v(\eta), \mathrm{diff}(\theta(\eta),\eta) = w(\eta), diff(v(\eta),\eta) + A.\big((f(\eta).v(\eta) - u(\eta)^2) - (\lambda).u(\eta)\big) - \mathrm{B.p.}\theta(\eta) = 0,\ \mathrm{diff}(w(\eta),\eta) + \left(\frac{Pr.(C)}{E}\right).(f(\eta).w(\eta) - u(\eta).\theta(\eta)) = 0\Big\}$;

$$ODE := \Big\{\frac{d}{d\eta} w(\eta) + 5.913158407\,(f(\eta).w(\eta)) - 5.913158407\,(u(\eta).\theta(\eta)) = 0, \frac{d}{d\eta} v(\eta) + 0.9908664759\,(f(\eta).v(\eta)) - 0.9908664759\,u(\eta)^2 - 0.09908664759\,u(\eta) - 0.9901571429\,\theta(\eta) = 0, \frac{d}{d\eta} f(\eta) = u(\eta), \frac{d}{d\eta}\theta(\eta) = w(\eta), \frac{d}{d\eta} u(\eta) = v(\eta)\Big\}$$

>$IC := \{f(0) = -S, u(0) = 1, \theta(0) = 1, v(0) = \alpha, w(0) = \beta\}$;

$$IC := \{f(0) = -0.1, \theta(0) = 1, u(0) = 1, v(0) = \alpha, w(0) = \beta\}$$

>$L := 1.5$;

$$L := 1.5$$

>$BC := \{u(L) = 0, \theta(L) = 0\}$;

$$BC := \{\theta(1.5) = 0, u(1.5) = 0\}$$

>$infolevel[shoot] := 1$:

>$S := shoot(ODE, IC, BC, FNS, [\alpha = -1.3502182232691166, \beta = -1.3069359569506])$:

shoot: Step # 1

shoot: Parameter values : alpha = -1.3502182232691166 beta = -1.3069359569506

shoot: Step # 2

shoot: Parameter values : alpha = HFloat(-1.4007364121558639) beta = HFloat(-2.204602887096368)

shoot: Step # 3

shoot: Parameter values : alpha = HFloat(-1.3578640057948173) beta = HFloat(-2.380078892014554)

shoot: Step # 4

shoot: Parameter values : alpha = HFloat(-1.3583310557121981) beta = HFloat(-2.378462988584847)

shoot: Step # 5

shoot: Parameter values : alpha = HFloat(-1.3583311255572168) beta = HFloat(-2.3784627502449682)

> u1 := odeplot(S, [η, u(η)], 0 ..L, color = red, linestyle = solid, *labels* = ["η", "f '(η)"], *labeldirections* = [*horizontal*, *vertical*], *labelfont* = [*timesnewroman*, 12]) :

> θ1 := odeplot(S, [η, θ(η)], 0 ..L, color = red, linestyle = solid, *labels* = ["η", "θ(η)"], *labeldirections* = [*horizontal*, *vertical*], *labelfont* = [*timesnewroman*, 12]) :

>$Pr := 6.2;\ S := 0.1;\ \Upsilon := 0.1;\ p := 1;\ \lambda := 0.1;$

$$Pr := 6.2$$

$$S := 0.1$$

$$\Upsilon := 0.1$$

$$p := 1$$

$$\lambda := 0.1$$

> $(\rho s) := 2600;\ (cps) := 425;\ (ks) := 6600;\ (\beta s) := 0.99\mathrm{E}-5;\ (\rho f) := 997;\ (cpf) := 4179;\ (kf) := 0.613;\ (\beta f) := 63\mathrm{E}-5;$

$$\rho s := 2600$$
$$cps := 425$$
$$ks := 6600$$
$$\beta s := 0.0000099$$
$$\rho f := 997$$
$$cpf := 4179$$
$$kf := 0.613$$
$$\beta f := 0.00063$$

> $(Kfn) := (ks) + 2.(kf) - 2.\Upsilon.((kf) - (ks));$

$$Kfn := 7921.1034$$

> $A := (1-\Upsilon)^{2.5}.\left(1 - \Upsilon + \Upsilon.\frac{(\rho s)}{(\rho f)}\right);$

$$A := 0.8919840084$$

> $C := \left(1 - \Upsilon + \Upsilon.\frac{(\rho s).(cps)}{(\rho f).(cpf)}\right);$

$$C := 0.9265212964$$

> $B := \left(1 - \Upsilon + \Upsilon.\frac{(\beta s)}{(\beta f)}\right);$

$$B := 0.9015714286$$

> $(Kf) := (ks) + 2.(kf) + 2.\Upsilon.((kf) - (ks));$

$$Kf := 5281.3486$$

> $E := \frac{Kfn}{Kf};$

$$E := 1.499825897$$

> $FNS := \{f(\eta), u(\eta), v(\eta), \theta(\eta), w(\eta)\}:$

> $ODE := \Big\{\mathrm{diff}(f(\eta), \eta) = u(\eta), \mathrm{diff}(u(\eta), \eta) = v(\eta), \mathrm{diff}(\theta(\eta), \eta) = w(\eta), diff(v(\eta), \eta) + A.((f(\eta).v(\eta) - u(\eta)^2) - (\lambda).u(\eta)) - \mathrm{B.p.}\theta(\eta) = 0,\ \mathrm{diff}(w(\eta), \eta) + \left(\frac{Pr.(C)}{E}\right).(f(\eta).w(\eta) - u(\eta).\theta(\eta)) = 0\Big\};$

$$ODE := \Big\{\frac{d}{d\eta} w(\eta) + 3.830065909\,(f(\eta).w(\eta)) - 3.830065909\,(u(\eta).\theta(\eta)) = 0,$$
$$\frac{d}{d\eta} v(\eta) + 0.8919840084\,(f(\eta).v(\eta)) - 0.8919840084\,u(\eta)^2 - 0.08919840084\,u(\eta)$$
$$- 0.9015714286\,\theta(\eta) = 0, \frac{d}{d\eta} f(\eta) = u(\eta), \frac{d}{d\eta}\theta(\eta) = w(\eta), \frac{d}{d\eta} u(\eta) = v(\eta)\Big\}$$

> $IC := \{f(0) = -S, u(0) = 1, \theta(0) = 1, v(0) = \alpha, w(0) = \beta\};$

$$IC := \{f(0) = -0.1, \theta(0) = 1, u(0) = 1, v(0) = \alpha, w(0) = \beta\}$$

> $L := 1.5;$

$$L := 1.5$$

>$BC := \{u(\mathrm{L}) = 0, \theta(\mathrm{L}) = 0\};$

$$BC := \{\theta(1.5) = 0, u(1.5) = 0\}$$

>$infolevel[shoot] := 1:$

>$S := shoot(ODE, IC, BC, FNS, [\alpha = -1.3502182232691166, \beta = -1.3069359569506]):$

shoot: Step # 1

shoot: Parameter values : alpha = -1.3502182232691166 beta = -1.3069359569506

shoot: Step # 2

shoot: Parameter values : alpha = HFloat(-1.3436642314225835) beta = HFloat(-1.8815399279074663)

shoot: Step # 3

shoot: Parameter values : alpha = HFloat(-1.3362255388372488) beta = HFloat(-1.913899355402682)

shoot: Step # 4

shoot: Parameter values : alpha = HFloat(-1.3362371826456134) beta = HFloat(-1.9138602021021935)

>

> $\mathrm{u2} := \mathrm{odeplot}(\mathrm{S}, [\eta, \mathrm{u}(\eta)], 0..\mathrm{L}, \mathrm{color} = \mathrm{green}, \mathrm{linestyle} = \mathrm{solid}, labels = ["\eta", "f'(\eta)"], labeldirections = [horizontal, vertical], labelfont = [timesnewroman, 12]):$

> $\theta 2 := \mathrm{odeplot}(\mathrm{S}, [\eta, \theta(\eta)], 0..\mathrm{L}, \mathrm{color} = \mathrm{green}, \mathrm{linestyle} = \mathrm{solid}, labels = ["\eta", "\theta(\eta)"], labeldirections = [horizontal, vertical], labelfont = [timesnewroman, 12]):$

>$Pr := 6.2;\ S := 0.1;\ \Upsilon := 0.2;\ p := 1;\ \lambda := 0.1;$

$$Pr := 6.2$$

$$S := 0.1$$

$$\Upsilon := 0.2$$

$$p := 1$$

$$\lambda := 0.1$$

> $(\rho s) := 2600;\ (cps) := 425;\ (ks) := 6600;\ (\beta s) := 0.99\mathrm{E}-5;\ (\rho f) := 997;\ (cpf) := 4179;\ (kf) := 0.613;\ (\beta f) := 63\mathrm{E}-5;$

$$\rho s := 2600$$

$$cps := 425$$

$$ks := 6600$$

$$\beta s := 0.0000099$$

$$\rho f := 997$$

$$cpf := 4179$$

$$kf := 0.613$$

$$\beta f := 0.00063$$

>$(Kfn) := (ks) + 2.(kf) - 2.\ \Upsilon.((kf) - (ks));$

$$Kfn := 9240.9808$$

>$A := (1 - \Upsilon)^{2.5}.\left(1 - \Upsilon + \Upsilon.\frac{(\rho s)}{(\rho f)}\right);$

$$A := 0.7565077740$$

>$C := \left(1 - \Upsilon + \Upsilon.\frac{(\rho s).(cps)}{(\rho f).(cpf)}\right);$

$$C := 0.8530425927$$

$>B := \left(1 - \Upsilon + \Upsilon.\frac{(\beta s)}{(\beta f)}\right);$

$$B := 0.8031428571$$

$>(Kf) := (ks) + 2.(kf) + 2.\,\Upsilon.((kf) - (ks));$

$$Kf := 3961.4712$$

$>E := \frac{Kfn}{Kf};$

$$E := 2.332714371$$

>

$>FNS := \{f(\eta), u(\eta), v(\eta), \theta(\eta), w(\eta)\} :$

$> ODE := \Big\{\mathrm{diff}(f(\eta), \eta) = u(\eta), \mathrm{diff}(u(\eta), \eta) = v(\eta), \mathrm{diff}(\theta(\eta), \eta) = w(\eta), diff(v(\eta), \eta)$
$+ A.\left(\left(f(\eta).v(\eta) - u(\eta)^2\right) - (\lambda).u(\eta)\right) - \mathrm{B.p.}\theta(\eta) = 0,\ \mathrm{diff}(w(\eta), \eta)$
$+ \left(\frac{Pr.(C)}{E}\right).(f(\eta).w(\eta) - u(\eta).\theta(\eta)) = 0\Big\};$

$$ODE := \Big\{\frac{d}{d\eta}\, w(\eta) + 2.267257467\,(f(\eta).w(\eta)) - 2.267257467\,(u(\eta).\theta(\eta)) = 0,$$
$$\frac{d}{d\eta}\, v(\eta) + 0.7565077740\,(f(\eta).v(\eta)) - 0.7565077740\, u(\eta)^2 - 0.07565077740\, u(\eta)$$
$$- 0.8031428571\,\theta(\eta) = 0, \frac{d}{d\eta} f(\eta) = u(\eta), \frac{d}{d\eta}\,\theta(\eta) = w(\eta), \frac{d}{d\eta}\, u(\eta) = v(\eta)\Big\}$$

$>IC := \{f(0) = -S, u(0) = 1, \theta(0) = 1, v(0) = \alpha, w(0) = \beta\};$

$$IC := \{f(0) = -0.1, \theta(0) = 1, u(0) = 1, v(0) = \alpha, w(0) = \beta\}$$

$>L := 1.5;$

$$L := 1.5$$

$>BC := \{u(\mathrm{L}) = 0, \theta(\mathrm{L}) = 0\};$

$$BC := \{\theta(1.5) = 0, u(1.5) = 0\}$$

$>infolevel[shoot] := 1 :$

$>S := shoot(ODE, IC, BC, FNS, [\alpha = -1.316979451166, \beta = -1.2041501906]) :$

shoot: Step # 1

shoot: Parameter values : alpha = -1.316979451166 beta = -1.2041501906

shoot: Step # 2

shoot: Parameter values : alpha = HFloat(-1.2928600037412683) beta = HFloat(-1.4860021718490022)

shoot: Step # 3

shoot: Parameter values : alpha = HFloat(-1.2924453079439482) beta = HFloat(-1.4880278574425372)

shoot: Step # 4

shoot: Parameter values : alpha = HFloat(-1.2924453333958885) beta = HFloat(-1.4880277786918374)

>

> u3 := odeplot(S, [η, u(η)], 0 ..L, color = black, linestyle = solid, *labels* = ["η", "f '(η)"], *labeldirections* = [*horizontal*, *vertical*], *labelfont* = [*timesnewroman*, 12]) :

$> \theta 3 := \mathrm{odeplot}(S, [\eta, \theta(\eta)], 0..L, \mathrm{color} = \mathrm{black}, \mathrm{linestyle} = \mathrm{solid}, labels = ["\eta", "\theta(\eta)"], labeldirections = [horizontal, vertical], labelfont = [timesnewroman, 12]) :$

$> Pr := 6.2;\ S := 0.1;\ \Upsilon := 0.01;\ p := 3;\ \lambda := 0.1;$

$$Pr := 6.2$$
$$S := 0.1$$
$$\Upsilon := 0.01$$
$$p := 3$$
$$\lambda := 0.1$$

$> (\rho s) := 2600;\ (cps) := 425;\ (ks) := 6600;\ (\beta s) := 0.99\mathrm{E}{-5};\ (\rho f) := 997;\ (cpf) := 4179;\ (kf) := 0.613;\ (\beta f) := 63\mathrm{E}{-5};$

$$\rho s := 2600$$
$$cps := 425$$
$$ks := 6600$$
$$\beta s := 0.0000099$$
$$\rho f := 997$$
$$cpf := 4179$$
$$kf := 0.613$$
$$\beta f := 0.00063$$

$> (Kfn) := (ks) + 2.(kf) - 2.\,\Upsilon.((kf) - (ks));$

$$Kfn := 6733.21374$$

$> A := (1 - \Upsilon)^{2.5}.\left(1 - \Upsilon + \Upsilon.\frac{(\rho s)}{(\rho f)}\right);$

$$A := 0.9908664759$$

$> C := \left(1 - \Upsilon + \Upsilon.\frac{(\rho s).(cps)}{(\rho f).(cpf)}\right);$

$$C := 0.9926521296$$

$> B := \left(1 - \Upsilon + \Upsilon.\frac{(\beta s)}{(\beta f)}\right);$

$$B := 0.9901571429$$

$> (Kf) := (ks) + 2.(kf) + 2.\,\Upsilon.((kf) - (ks));$

$$Kf := 6469.23826$$

$> E := \frac{Kfn}{Kf};$

$$E := 1.040804724$$

$>$

$> FNS := \{f(\eta), u(\eta), v(\eta), \theta(\eta), w(\eta)\} :$

> $ODE := \{\mathrm{diff}(f(\eta),\eta) = u(\eta), \mathrm{diff}(u(\eta),\eta) = v(\eta), \mathrm{diff}(\theta(\eta),\eta) = w(\eta), diff(v(\eta),\eta) + A.((f(\eta).v(\eta) - u(\eta)^2) - (\lambda).u(\eta)) - \mathrm{B.p.}\theta(\eta) = 0,\ \mathrm{diff}(w(\eta),\eta) + \left(\frac{Pr.(C)}{E}\right).(f(\eta).w(\eta) - u(\eta).\theta(\eta)) = 0\};$

$$ODE := \left\{\frac{d}{d\eta} w(\eta) + 5.913158407\,(f(\eta).w(\eta)) - 5.913158407\,(u(\eta).\theta(\eta)) = 0, \frac{d}{d\eta} v(\eta) + 0.9908664759\,(f(\eta).v(\eta)) - 0.9908664759\,u(\eta)^2 - 0.09908664759\,u(\eta) - 2.970471429\,\theta(\eta) = 0, \frac{d}{d\eta} f(\eta) = u(\eta), \frac{d}{d\eta}\theta(\eta) = w(\eta), \frac{d}{d\eta} u(\eta) = v(\eta)\right\}$$

> $IC := \{f(0) = -S, u(0) = 1, \theta(0) = 1, v(0) = \alpha, w(0) = \beta\};$

$$IC := \{f(0) = -0.1, \theta(0) = 1, u(0) = 1, v(0) = \alpha, w(0) = \beta\}$$

> $L := 1.5;$

$$L := 1.5$$

> $BC := \{u(\mathrm{L}) = 0, \theta(\mathrm{L}) = 0\};$

$$BC := \{\theta(1.5) = 0, u(1.5) = 0\}$$

> $infolevel[shoot] := 1:$

> $S := shoot(ODE, IC, BC, FNS, [\alpha = -1.870320872266, \beta = -1.33608848101906]):$

shoot: Step # 1

shoot: Parameter values : alpha = -1.870320872266 beta = -1.33608848101906

shoot: Step # 2

shoot: Parameter values : alpha = HFloat(-2.151578021337767) beta = HFloat(-1.8655325238229978)

shoot: Step # 3

shoot: Parameter values : alpha = HFloat(-1.8553384130901625) beta = HFloat(-2.307510455025882)

shoot: Step # 4

shoot: Parameter values : alpha = HFloat(-1.8926745483540834) beta = HFloat(-2.2566616750699993)

shoot: Step # 5

shoot: Parameter values : alpha = HFloat(-1.893273407372343) beta = HFloat(-2.2558612760075882)

shoot: Step # 6

shoot: Parameter values : alpha = HFloat(-1.893273568241907) beta = HFloat(-2.2558610610186904)

> u4 := odeplot(S, [η, u(η)], 0 ..L, color = red, linestyle = dash, *labels* = ["η", "f '(η)"], *labeldirections* = [*horizontal*, *vertical*], *labelfont* = [*timesnewroman*, 12]) :

> θ4 := odeplot(S, [η, θ(η)], 0 ..L, color = red, linestyle = dash, *labels* = ["η", "θ(η)"], *labeldirections* = [*horizontal*, *vertical*], *labelfont* = [*timesnewroman*, 12]) :

> $Pr := 6.2;\ S := 0.1;\ \Upsilon := 0.1;\ p := 3;\ \lambda := 0.1;$

$$Pr := 6.2$$

$$S := 0.1$$

$$\Upsilon := 0.1$$

$$p := 3$$

$$\lambda := 0.1$$

> $(\rho s) := 2600;\ (cps) := 425;\ (ks) := 6600;\ (\beta s) := 0.99\mathrm{E}{-5};\ (\rho f) := 997;\ (cpf) := 4179;$
$(kf) := 0.613;\ (\beta f) := 63\mathrm{E}{-5};$

$$\rho s := 2600$$
$$cps := 425$$
$$ks := 6600$$
$$\beta s := 0.0000099$$
$$\rho f := 997$$
$$cpf := 4179$$
$$kf := 0.613$$
$$\beta f := 0.00063$$

> $(Kfn) := (ks) + 2.(kf) - 2.\Upsilon.((kf) - (ks));$

$$Kfn := 7921.1034$$

> $A := (1-\Upsilon)^{2.5}.\left(1 - \Upsilon + \Upsilon.\frac{(\rho s)}{(\rho f)}\right);$

$$A := 0.8919840084$$

> $C := \left(1 - \Upsilon + \Upsilon.\frac{(\rho s).(cps)}{(\rho f).(cpf)}\right);$

$$C := 0.9265212964$$

> $B := \left(1 - \Upsilon + \Upsilon.\frac{(\beta s)}{(\beta f)}\right);$

$$B := 0.9015714286$$

> $(Kf) := (ks) + 2.(kf) + 2.\Upsilon.((kf) - (ks));$

$$Kf := 5281.3486$$

> $E := \frac{Kfn}{Kf};$

$$E := 1.499825897$$

>

> $FNS := \{f(\eta), u(\eta), v(\eta), \theta(\eta), w(\eta)\}:$

> $ODE := \Big\{\mathrm{diff}(f(\eta),\eta) = u(\eta), \mathrm{diff}(u(\eta),\eta) = v(\eta), \mathrm{diff}(\theta(\eta),\eta) = w(\eta), diff(v(\eta),\eta) + A.((f(\eta).v(\eta) - u(\eta)^2) - (\lambda).u(\eta)) - \mathrm{B.p.}\theta(\eta) = 0,\ \mathrm{diff}(w(\eta),\eta) + \left(\frac{Pr.(C)}{E}\right).(f(\eta).w(\eta) - u(\eta).\theta(\eta)) = 0\Big\};$

$$ODE := \Big\{\frac{d}{d\eta} w(\eta) + 3.830065909\,(f(\eta).w(\eta)) - 3.830065909\,(u(\eta).\theta(\eta)) = 0, \frac{d}{d\eta} v(\eta) + 0.8919840084\,(f(\eta).v(\eta)) - 0.8919840084\,u(\eta)^2 - 0.08919840084\,u(\eta) - 2.704714286\,\theta(\eta) = 0, \frac{d}{d\eta} f(\eta) = u(\eta), \frac{d}{d\eta}\theta(\eta) = w(\eta), \frac{d}{d\eta} u(\eta) = v(\eta)\Big\}$$

> $IC := \{f(0) = -S, u(0) = 1, \theta(0) = 1, v(0) = \alpha, w(0) = \beta\};$

$$IC := \{f(0) = -0.1, \theta(0) = 1, u(0) = 1, v(0) = \alpha, w(0) = \beta\}$$

$>L := 1.5;$

$$L := 1.5$$

$>BC := \{u(\mathrm{L}) = 0, \theta(\mathrm{L}) = 0\};$

$$BC := \{\theta(1.5) = 0, u(1.5) = 0\}$$

$>infolevel[shoot] := 1 :$

$>S := shoot(ODE, IC, BC, FNS, [\alpha = -1.803678674266, \beta = -1.24672016701906]) :$

shoot: Step # 1

shoot: Parameter values : alpha = -1.803678674266 beta = -1.24672016701906

shoot: Step # 2

shoot: Parameter values : alpha = HFloat(-1.9885664587864982) beta = HFloat(-1.6587124595445737)

shoot: Step # 3

shoot: Parameter values : alpha = HFloat(-1.907763847137651) beta = HFloat(-1.789557255233261)

shoot: Step # 4

shoot: Parameter values : alpha = HFloat(-1.9096934551644011) beta = HFloat(-1.7870261659884592)

shoot: Step # 5

shoot: Parameter values : alpha = HFloat(-1.9096948451710412) beta = HFloat(-1.7870243481142358)

>

> u5 := odeplot(S, [η, u(η)], 0 ..L, color = green, linestyle = dash, *labels* = ["η", "f '(η)"],
labeldirections = [*horizontal*, *vertical*], *labelfont* = [*timesnewroman*, 12]) :

> θ5 := odeplot(S, [η, θ(η)], 0 ..L, color = green, linestyle = dash, *labels* = ["η", "θ(η)"],
labeldirections = [*horizontal*, *vertical*], *labelfont* = [*timesnewroman*, 12]) :

$>Pr := 6.2;\ S := 0.1;\ \Upsilon := 0.2;\ p := 3;\ \lambda := 0.1;$

$$Pr := 6.2$$

$$S := 0.1$$

$$\Upsilon := 0.2$$

$$p := 3$$

$$\lambda := 0.1$$

$> (\rho s) := 2600;\ (cps) := 425;\ (ks) := 6600;\ (\beta s) := 0.99\mathrm{E}-5;\ (\rho f) := 997;\ (cpf) := 4179;$
$(kf) := 0.613;\ (\beta f) := 63\mathrm{E}-5;$

$$\rho s := 2600$$

$$cps := 425$$

$$ks := 6600$$

$$\beta s := 0.0000099$$

$$\rho f := 997$$

$$cpf := 4179$$

$$kf := 0.613$$

$$\beta f := 0.00063$$

$>(Kfn) := (ks) + 2.(kf) - 2.\ \Upsilon.((kf) - (ks));$

$$Kfn := 9240.9808$$

$$>A := (1 - \Upsilon)^{2.5}.\left(1 - \Upsilon + \Upsilon.\frac{(\rho s)}{(\rho f)}\right);$$

$$A := 0.7565077740$$

$$>C := \left(1 - \Upsilon + \Upsilon.\frac{(\rho s).(cps)}{(\rho f).(cpf)}\right);$$

$$C := 0.8530425927$$

$$>\mathbf{E} := \left(\mathbf{1} - \Upsilon + \Upsilon.\frac{(\beta s)}{(\beta f)}\right);$$

$$E := 0.8031428571$$

$$>(Kf) := (ks) + 2.(kf) + 2.\,\Upsilon.((kf) - (ks));$$

$$Kf := 3961.4712$$

$$>E := \frac{Kfn}{Kf};$$

$$E := 2.332714371$$

\>

$$>FNS := \{f(\eta), u(\eta), v(\eta), \theta(\eta), w(\eta)\}:$$

$$> ODE := \Big\{\mathrm{diff}(f(\eta),\eta) = u(\eta), \mathrm{diff}(u(\eta),\eta) = v(\eta), \mathrm{diff}(\theta(\eta),\eta) = w(\eta), diff(v(\eta),\eta) + A.\left(\left(f(\eta).v(\eta) - u(\eta)^2\right) - (\lambda).u(\eta)\right) - \mathrm{B.p.}\theta(\eta) = 0,\ \mathrm{diff}(w(\eta),\eta) + \left(\frac{Pr.(C)}{E}\right).(f(\eta).w(\eta) - u(\eta).\theta(\eta)) = 0\Big\};$$

$$ODE := \Big\{\frac{\mathrm{d}}{\mathrm{d}\eta}w(\eta) + 2.267257467\,(f(\eta).w(\eta)) - 2.267257467\,(u(\eta).\theta(\eta)) = 0, \frac{\mathrm{d}}{\mathrm{d}\eta}v(\eta) + 0.7565077740\,(f(\eta).v(\eta)) - 0.7565077740\,u(\eta)^2 - 0.07565077740\,u(\eta) - 2.704714286\,\theta(\eta) = 0, \frac{\mathrm{d}}{\mathrm{d}\eta}f(\eta) = u(\eta), \frac{\mathrm{d}}{\mathrm{d}\eta}\theta(\eta) = w(\eta), \frac{\mathrm{d}}{\mathrm{d}\eta}u(\eta) = v(\eta)\Big\}$$

$$>IC := \{f(0) = -S, u(0) = 1, \theta(0) = 1, v(0) = \alpha, w(0) = \beta\};$$

$$IC := \{f(0) = -0.1, \theta(0) = 1, u(0) = 1, v(0) = \alpha, w(0) = \beta\}$$

$$>L := 1.5;$$

$$L := 1.5$$

$$>BC := \{u(\mathrm{L}) = 0, \theta(\mathrm{L}) = 0\};$$

$$BC := \{\theta(1.5) = 0, u(1.5) = 0\}$$

$$>infolevel[shoot] := 1:$$

$$>S := shoot(ODE, IC, BC, FNS, [\alpha = -1.740046944601121166, \beta = -1.1630810581906]):$$

shoot: Step # 1

shoot: Parameter values : alpha = -1.740046944601121166 beta = -1.1630810581906

shoot: Step # 2

shoot: Parameter values : alpha = HFloat(-2.0050853591445925) beta = HFloat(-1.3346976346929176)

shoot: Step # 3

shoot: Parameter values : alpha = HFloat(-1.9973938068123533) beta = HFloat(-1.356889733128276)

shoot: Step # 4

shoot: Parameter values : alpha = HFloat(-1.9973704361216882) beta = HFloat(-1.35692510363633)

\>

> u6 := odeplot(S, [η, u(η)], 0 ..L, color = black, linestyle = dash, *labels* = ["η", "f '(η)"], *labeldirections* = [*horizontal*, *vertical*], *labelfont* = [*timesnewroman*, 12]) :

> θ6 := odeplot(S, [η, θ(η)], 0 ..L, color = black, linestyle = dash, *labels* = ["η", "θ(η)"], *labeldirections* = [*horizontal*, *vertical*], *labelfont* = [*timesnewroman*, 12]) :

>$Pr := 6.2;\ S := 0.1;\ \Upsilon := 0.01;\ p := 1;\ \lambda := 0.1;$

$$Pr := 6.2$$
$$S := 0.1$$
$$\Upsilon := 0.01$$
$$p := 1$$
$$\lambda := 0.1$$

> $(\rho s) := 8933;\ (cps) := 385;\ (ks) := 401;\ (\beta s) := 4.89\mathrm{E}{-5};\ (\rho f) := 997;\ (cpf) := 4179;\ (kf) := 0.613;\ (\beta f) := 63\mathrm{E}{-5};$

$$\rho s := 8933$$
$$cps := 385$$
$$ks := 401$$
$$\beta s := 0.0000489$$
$$\rho f := 997$$
$$cpf := 4179$$
$$kf := 0.613$$
$$\beta f := 0.00063$$

>$(Kfn) := (ks) + 2.(kf) - 2.\,\Upsilon.((kf) - (ks));$

$$Kfn := 410.23374$$

>$A := (1 - \Upsilon)^{2.5}.\left(1 - \Upsilon + \Upsilon.\frac{(\rho s)}{(\rho f)}\right);$

>$C := \left(1 - \Upsilon + \Upsilon.\frac{(\rho s).(cps)}{(\rho f).(cpf)}\right);$

$$C := 0.9982544955$$

>$B := \left(1 - \Upsilon + \Upsilon.\frac{(\beta s)}{(\beta f)}\right);$

$$B := 0.9907761905$$

>$(Kf) := (ks) + 2.(kf) + 2.\,\Upsilon.((kf) - (ks));$

$$Kf := 394.21826$$

>$E := \frac{Kfn}{Kf};$

$$E := 1.040625921$$

>

>$FNS := \{f(\eta), u(\eta), v(\eta), \theta(\eta), w(\eta)\} :$

> $ODE := \{\mathrm{diff}(f(\eta), \eta) = u(\eta), \mathrm{diff}(u(\eta), \eta) = v(\eta), \mathrm{diff}(\theta(\eta), \eta) = w(\eta), diff(v(\eta), \eta) + A.((f(\eta).v(\eta) - u(\eta)^2) - \lambda.u(\eta)) - \mathrm{B.p.}\theta(\eta) = 0, \ \mathrm{diff}(w(\eta), \eta) + \left(\frac{Pr.(C)}{E}\right).(f(\eta).w(\eta) - u(\eta).\theta(\eta)) = 0\};$

$$ODE := \left\{\frac{\mathrm{d}}{\mathrm{d}\eta} w(\eta) + 5.947553052\,(f(\eta).w(\eta)) - 5.947553052\,(u(\eta).\theta(\eta)) = 0, \frac{\mathrm{d}}{\mathrm{d}\eta} v(\eta) + 1.052810913\,(f(\eta).v(\eta)) - 1.052810913\,u(\eta)^2 - 0.1052810913\,u(\eta) - 0.9907761905\,\theta(\eta) = 0, \frac{\mathrm{d}}{\mathrm{d}\eta} f(\eta) = u(\eta), \frac{\mathrm{d}}{\mathrm{d}\eta}\theta(\eta) = w(\eta), \frac{\mathrm{d}}{\mathrm{d}\eta} u(\eta) = v(\eta)\right\}$$

> $IC := \{f(0) = -S, u(0) = 1, \theta(0) = 1, v(0) = \alpha, w(0) = \beta\};$

$$IC := \{f(0) = -0.1, \theta(0) = 1, u(0) = 1, v(0) = \alpha, w(0) = \beta\}$$

> $L := 1.5;$

$$L := 1.5$$

> $BC := \{u(\mathrm{L}) = 0, \theta(\mathrm{L}) = 0\};$

$$BC := \{\theta(1.5) = 0, u(1.5) = 0\}$$

> $infolevel[shoot] := 1:$

> $S := shoot(ODE, IC, BC, FNS, [\alpha = -1.3802491292691166, \beta = -1.41527269969506]):$

shoot: Step # 1

shoot: Parameter values : alpha = -1.3802491292691166 beta = -1.41527269969506

shoot: Step # 2

shoot: Parameter values : alpha = HFloat(-1.4136857639706875) beta = HFloat(-2.2404933495594546)

shoot: Step # 3

shoot: Parameter values : alpha = HFloat(-1.379578852889546) beta = HFloat(-2.3810089147856224)

shoot: Step # 4

shoot: Parameter values : alpha = HFloat(-1.3798851400304155) beta = HFloat(-2.3799546399504736)

shoot: Step # 5

shoot: Parameter values : alpha = HFloat(-1.379885171344507) beta = HFloat(-2.379954533306278)

>

> u7 := odeplot(S, [η, u(η)], 0 ..L, color = red, linestyle = solid, *labels* = ["η", "f '(η)"], *labeldirections* = [*horizontal*, *vertical*], *labelfont* = [*timesnewroman*, 12]) :

> θ7 := odeplot(S, [η, θ(η)], 0 ..L, color = red, linestyle = solid, *labels* = ["η", "θ(η)"], *labeldirections* = [*horizontal*, *vertical*], *labelfont* = [*timesnewroman*, 12]) :

> $Pr := 6.2;\ S := 0.1;\ \Upsilon := 0.1;\ p := 1;\ \lambda := 0.1;$

$$Pr := 6.2$$

$$S := 0.1$$

$$\Upsilon := 0.1$$

$$p := 1$$

$$\lambda := 0.1$$

$> (\rho s) := 8933;\ (cps) := 385;\ (ks) := 401;\ (\beta s) := 4.89\mathrm{E}{-5};\ (\rho f) := 997;\ (cpf) := 4179;\ (kf) := 0.613;\ (\beta f) := 63\mathrm{E}{-5};$

$$\rho s := 8933$$
$$cps := 385$$
$$ks := 401$$
$$\beta s := 0.0000489$$
$$\rho f := 997$$
$$cpf := 4179$$
$$kf := 0.613$$
$$\beta f := 0.00063$$

$>(Kfn) := (ks) + 2.(kf) - 2.\Upsilon.((kf) - (ks));$

$$Kfn := 482.3034$$

$>A := (1-\Upsilon)^{2.5}.\left(1-\Upsilon+\Upsilon.\frac{(\rho s)}{(\rho f)}\right);$

$$A := 1.380097266$$

$>C := \left(1-\Upsilon+\Upsilon.\frac{(\rho s).(cps)}{(\rho f).(cpf)}\right);$

$$C := 0.9825449548$$

$>B := \left(1-\Upsilon+\Upsilon.\frac{(\beta s)}{(\beta f)}\right);$

$$B := 0.9077619048$$

$>(Kf) := (ks) + 2.(kf) + 2.\Upsilon.((kf) - (ks));$

$$Kf := 322.1486$$

$>E := \frac{Kfn}{Kf};$

$$E := 1.497145727$$

$>$

$>FNS := \{f(\eta), u(\eta), v(\eta), \theta(\eta), w(\eta)\}:$

$> ODE := \Big\{\mathrm{diff}(f(\eta),\eta) = u(\eta), \mathrm{diff}(u(\eta),\eta) = v(\eta), \mathrm{diff}(\theta(\eta),\eta) = w(\eta), \mathit{diff}(v(\eta),\eta) + A.((f(\eta).v(\eta) - u(\eta)^2) - (\lambda).u(\eta)) - \mathrm{B.p.}\theta(\eta) = 0,\ \mathrm{diff}(w(\eta),\eta) + \left(\frac{Pr.(C)}{E}\right).(f(\eta).w(\eta) - u(\eta).\theta(\eta)) = 0\Big\};$

$$ODE := \left\{\frac{d}{d\eta}w(\eta) + 4.068928368\,(f(\eta).w(\eta)) - 4.068928368\,(u(\eta).\theta(\eta)) = 0, \frac{d}{d\eta}v(\eta) + 1.380097266\,(f(\eta).v(\eta)) - 1.380097266\,u(\eta)^2 - 0.1380097266\,u(\eta) - 0.9077619048\,\theta(\eta) = 0, \frac{d}{d\eta}f(\eta) = u(\eta), \frac{d}{d\eta}\theta(\eta) = w(\eta), \frac{d}{d\eta}u(\eta) = v(\eta)\right\}$$

$>IC := \{f(0) = -S, u(0) = 1, \theta(0) = 1, v(0) = \alpha, w(0) = \beta\};$

$$IC := \{f(0) = -0.1, \theta(0) = 1, u(0) = 1, v(0) = \alpha, w(0) = \beta\}$$

$>L := 1.5;$

$$L := 1.5$$

$>BC := \{u(\mathrm{L}) = 0, \theta(\mathrm{L}) = 0\};$

$$BC := \{\theta(1.5) = 0, u(1.5) = 0\}$$

$>infolevel[shoot] := 1 :$

$>S := shoot(ODE, IC, BC, FNS, [\alpha = -1.38146851166, \beta = -1.3177369506]) :$

shoot: Step # 1

shoot: Parameter values : alpha = -1.38146851166 beta = -1.3177369506

shoot: Step # 2

shoot: Parameter values : alpha = HFloat(-1.510521360030598) beta = HFloat(-1.8593131528665499)

shoot: Step # 3

shoot: Parameter values : alpha = HFloat(-1.4987088588953155) beta = HFloat(-1.9356243486234501)

shoot: Step # 4

shoot: Parameter values : alpha = HFloat(-1.4987149295076436) beta = HFloat(-1.9356385510633154)

\>

$>$ u8 := odeplot(S, [η, u(η)], 0 ..L, color = green, linestyle = solid, *labels* = ["η", "f '(η)"], *labeldirections* = [*horizontal*, *vertical*], *labelfont* = [*timesnewroman*, 12]) :

$>$ θ8 := odeplot(S, [η, θ(η)], 0 ..L, color = green, linestyle = solid, *labels* = ["η", "θ(η)"], *labeldirections* = [*horizontal*, *vertical*], *labelfont* = [*timesnewroman*, 12]) :

$>Pr := 6.2;\ S := 0.1;\ \Upsilon := 0.2;\ p := 1;\ \lambda := 0.1;$

$$Pr := 6.2$$

$$S := 0.1$$

$$\Upsilon := 0.2$$

$$p := 1$$

$$\lambda := 0.1$$

$> (\rho s) := 8933;\ (cps) := 385;\ (ks) := 401;\ (\beta s) := 4.89\mathrm{E}-5;\ (\rho f) := 997;\ (cpf) := 4179;\ (kf) := 0.613;\ (\beta f) := 63\mathrm{E}-5;$

$$\rho s := 8933$$

$$cps := 385$$

$$ks := 401$$

$$\beta s := 0.0000489$$

$$\rho f := 997$$

$$cpf := 4179$$

$$kf := 0.613$$

$$\beta f := 0.00063$$

$>(Kfn) := (ks) + 2.(kf) - 2.\ \Upsilon.((kf) - (ks));$

$$Kfn := 562.3808$$

$>A := (1 - \Upsilon)^{2.5}.\left(1 - \Upsilon + \Upsilon.\frac{(\rho s)}{(\rho f)}\right);$

$$A := 1.483733599$$

$$>C := \left(1 - \Upsilon + \Upsilon.\frac{(\rho s).(cps)}{(\rho f).(cpf)}\right);$$

$$C := 0.9650899096$$

$$>B := \left(1 - \Upsilon + \Upsilon.\frac{(\beta s)}{(\beta f)}\right);$$

$$B := 0.8155238095$$

$$>(Kf) := (ks) + 2.(kf) + 2.\,\Upsilon.((kf) - (ks));$$

$$Kf := 242.0712$$

$$>E := \frac{Kfn}{Kf};$$

$$E := 2.323204082$$

>

$$>FNS := \{f(\eta), u(\eta), v(\eta), \theta(\eta), w(\eta)\} :$$

$$> ODE := \Big\{\mathrm{diff}(f(\eta), \eta) = u(\eta), \mathrm{diff}(u(\eta), \eta) = v(\eta), \mathrm{diff}(\theta(\eta), \eta) = w(\eta), diff(v(\eta), \eta) + A.\left(\left(f(\eta).v(\eta) - u(\eta)^2\right) - (\lambda).u(\eta)\right) - \mathrm{B.p.}\theta(\eta) = 0,\ \mathrm{diff}(w(\eta), \eta) + \left(\frac{Pr.(C)}{E}\right).(f(\eta).w(\eta) - u(\eta).\theta(\eta)) = 0\Big\};$$

$$ODE := \Big\{\frac{\mathrm{d}}{\mathrm{d}\eta} w(\eta) + 2.575562554\,(f(\eta).w(\eta)) - 2.575562554\,(u(\eta).\theta(\eta)) = 0, \frac{\mathrm{d}}{\mathrm{d}\eta} v(\eta) + 1.483733599\,(f(\eta).v(\eta)) - 1.483733599\,u(\eta)^2 - 0.1483733599\,u(\eta) - 0.8155238095\,\theta(\eta) = 0, \frac{\mathrm{d}}{\mathrm{d}\eta} f(\eta) = u(\eta), \frac{\mathrm{d}}{\mathrm{d}\eta} \theta(\eta) = w(\eta), \frac{\mathrm{d}}{\mathrm{d}\eta} u(\eta) = v(\eta)\Big\}$$

$$>IC := \{f(0) = -S, u(0) = 1, \theta(0) = 1, v(0) = \alpha, w(0) = \beta\};$$

$$IC := \{f(0) = -0.1, \theta(0) = 1, u(0) = 1, v(0) = \alpha, w(0) = \beta\}$$

$$>L := 1.5;$$

$$L := 1.5$$

$$>BC := \{u(\mathrm{L}) = 0, \theta(\mathrm{L}) = 0\};$$

$$BC := \{\theta(1.5) = 0, u(1.5) = 0\}$$

$$>infolevel[shoot] := 1 :$$

$$>S := shoot(ODE, IC, BC, FNS, [\alpha = -1.366542151166, \beta = -1.222872163801906]) :$$

shoot: Step # 1

shoot: Parameter values : alpha = -1.366542151166 beta = -1.222872163801906

shoot: Step # 2

shoot: Parameter values : alpha = HFloat(-1.5325087411281795) beta = HFloat(-1.5114100794931464)

shoot: Step # 3

shoot: Parameter values : alpha = HFloat(-1.5339159150943094) beta = HFloat(-1.5350195528749055)

shoot: Step # 4

shoot: Parameter values : alpha = HFloat(-1.5339079899348849) beta = HFloat(-1.5350702716222424)

> $u9 := \text{odeplot}(S, [\eta, u(\eta)], 0..L, \text{color} = \text{black}, \text{linestyle} = \text{solid}, labels = ["\eta", "f'(\eta)"], labeldirections = [horizontal, vertical], labelfont = [timesnewroman, 12])$:

> $\theta 9 := \text{odeplot}(S, [\eta, \theta(\eta)], 0..L, \text{color} = \text{black}, \text{linestyle} = \text{solid}, labels = ["\eta", "\theta(\eta)"], labeldirections = [horizontal, vertical], labelfont = [timesnewroman, 12])$:

> $Pr := 6.2;\ S := 0.1;\ \Upsilon := 0.01;\ p := 3;\ \lambda := 0, 1;$

$$Pr := 6.2$$
$$S := 0.1$$
$$\Upsilon := 0.01$$
$$p := 3$$
$$\lambda := 0, 1$$

> $(\rho s) := 8933;\ (cps) := 385;\ (ks) := 401;\ (\beta s) := 4.89\text{E}-5;\ (\rho f) := 997;\ (cpf) := 4179;\ (kf) := 0.613;\ (\beta f) := 63\text{E}-5;$

$$\rho s := 8933$$
$$cps := 385$$
$$ks := 401$$
$$\beta s := 0.0000489$$
$$\rho f := 997$$
$$cpf := 4179$$
$$kf := 0.613$$
$$\beta f := 0.00063$$

> $(Kfn) := (ks) + 2.(kf) - 2.\,\Upsilon.((kf) - (ks));$

$$Kfn := 410.23374$$

> $A := (1-\Upsilon)^{2.5}.\left(1 - \Upsilon + \Upsilon.\frac{(\rho s)}{(\rho f)}\right);$

$$A := 1.052810913$$

> $C := \left(1 - \Upsilon + \Upsilon.\frac{(\rho s).(cps)}{(\rho f).(cpf)}\right);$

$$C := 0.9982544955$$

> $B := \left(1 - \Upsilon + \Upsilon.\frac{(\beta s)}{(\beta f)}\right);$

$$B := 0.9907761905$$

> $(Kf) := (ks) + 2.(kf) + 2.\,\Upsilon.((kf) - (ks));$

$$Kf := 394.21826$$

> $E := \frac{Kfn}{Kf};$

$$E := 1.040625921$$

>

> $FNS := \{f(\eta), u(\eta), v(\eta), \theta(\eta), w(\eta)\}$:

> $ODE := \{\mathrm{diff}(f(\eta),\eta) = u(\eta), \mathrm{diff}(u(\eta),\eta) = v(\eta), \mathrm{diff}(\theta(\eta),\eta) = w(\eta), diff(v(\eta),\eta) + A.((f(\eta).v(\eta) - u(\eta)^2) - (\lambda).u(\eta)) - \mathrm{B.p.}\theta(\eta) = 0,\ \mathrm{diff}(w(\eta),\eta) + \left(\frac{Pr.(C)}{E}\right).(f(\eta).w(\eta) - u(\eta).\theta(\eta)) = 0\};$

$$ODE := \left\{\frac{\mathrm{d}}{\mathrm{d}\eta} w(\eta) + 5.947553052\,(f(\eta).w(\eta)) - 5.947553052\,(u(\eta).\theta(\eta)) = 0, \frac{\mathrm{d}}{\mathrm{d}\eta} v(\eta) + 1.052810913\,(f(\eta).v(\eta)) - 1.052810913\,u(\eta)^2 - 2.972328572\,\theta(\eta) = 0, \frac{\mathrm{d}}{\mathrm{d}\eta} f(\eta) = u(\eta), \frac{\mathrm{d}}{\mathrm{d}\eta}\theta(\eta) = w(\eta), \frac{\mathrm{d}}{\mathrm{d}\eta} u(\eta) = v(\eta)\right\}$$

>$IC := \{f(0) = -S, u(0) = 1, \theta(0) = 1, v(0) = \alpha, w(0) = \beta\};$

$$IC := \{f(0) = -0.1, \theta(0) = 1, u(0) = 1, v(0) = \alpha, w(0) = \beta\}$$

>$L := 1.5;$

$$L := 1.5$$

>$BC := \{u(\mathrm{L}) = 0, \theta(\mathrm{L}) = 0\};$

$$BC := \{\theta(1.5) = 0, u(1.5) = 0\}$$

>$infolevel[shoot] := 1:$

>$S := shoot(ODE, IC, BC, FNS, [\alpha = -2.330463324266, \beta = -1.109894751906]):$

shoot: Step # 1

shoot: Parameter values : alpha = -2.330463324266 beta = -1.109894751906

shoot: Step # 2

shoot: Parameter values : alpha = HFloat(-2.0779690586958512) beta = HFloat(-1.9905897054041963)

shoot: Step # 3

shoot: Parameter values : alpha = HFloat(-1.8502257974447782) beta = HFloat(-2.301064855156073)

shoot: Step # 4

shoot: Parameter values : alpha = HFloat(-1.8750997360452377) beta = HFloat(-2.268091684957663)

shoot: Step # 5

shoot: Parameter values : alpha = HFloat(-1.875377944597948) beta = HFloat(-2.267727110648955)

shoot: Step # 6

shoot: Parameter values : alpha = HFloat(-1.8753779802870238) beta = HFloat(-2.2677270638655136)

>-1.4682614

$$-1.4682614$$

> u10 := odeplot(S, [η, u(η)], 0 ..L, color = red, linestyle = dash, *labels* = ["η", "f '(η)"], *labeldirections* = [*horizontal*, *vertical*], *labelfont* = [*timesnewroman*, 12]) :

> θ10 := odeplot(S, [η, θ(η)], 0 ..L, color = red, linestyle = dash, *labels* = ["η", "θ(η)"], *labeldirections* = [*horizontal*, *vertical*], *labelfont* = [*timesnewroman*, 12]) :

>$Pr := 6.2;\ S := 0.1;\ \Upsilon := 0.1;\ p := 3;\ \lambda := 0.1;$

$$Pr := 6.2$$

$$S := 0.1$$

$$\Upsilon := 0.1$$

$$p := 3$$

$$\lambda := 0.1$$

> $(\rho s) := 8933;\ (cps) := 385;\ (ks) := 401;\ (\beta s) := 4.89\mathrm{E}{-5};\ (\rho f) := 997;\ (cpf) := 4179;\ (kf) := 0.613;\ (\beta f) := 63\mathrm{E}{-5};$

$$\rho s := 8933$$

$$cps := 385$$

$$ks := 401$$

$$\beta s := 0.0000489$$

$$\rho f := 997$$

$$cpf := 4179$$

$$kf := 0.613$$

$$\beta f := 0.00063$$

> $(Kfn) := (ks) + 2.(kf) - 2.\Upsilon.((kf) - (ks));$

$$Kfn := 482.3034$$

> $A := (1-\Upsilon)^{2.5}.\left(1 - \Upsilon + \Upsilon.\frac{(\rho s)}{(\rho f)}\right);$

$$A := 1.380097266$$

> $C := \left(1 - \Upsilon + \Upsilon.\frac{(\rho s).(cps)}{(\rho f).(cpf)}\right);$

$$C := 0.9825449548$$

> $B := \left(1 - \Upsilon + \Upsilon.\frac{(\beta s)}{(\beta f)}\right);$

$$B := 0.9077619048$$

> $(Kf) := (ks) + 2.(kf) + 2.\Upsilon.((kf) - (ks));$

$$Kf := 322.1486$$

> $E := \frac{Kfn}{Kf};$

$$E := 1.497145727$$

>

> $FNS := \{f(\eta), u(\eta), v(\eta), \theta(\eta), w(\eta)\}:$

> $ODE := \Big\{\mathrm{diff}(f(\eta), \eta) = u(\eta), \mathrm{diff}(u(\eta), \eta) = v(\eta), \mathrm{diff}(\theta(\eta), \eta) = w(\eta), diff(v(\eta), \eta) + A.\big((f(\eta).v(\eta) - u(\eta)^2) - (\lambda).u(\eta)\big) - \mathrm{B.p.}\theta(\eta) = 0,\ \mathrm{diff}(w(\eta), \eta) + \left(\frac{Pr.(C)}{E}\right).(f(\eta).w(\eta) - u(\eta).\theta(\eta)) = 0\Big\};$

$$ODE := \left\{ \frac{d}{d\eta} w(\eta) + 4.068928368\,(f(\eta).w(\eta)) - 4.068928368\,(u(\eta).\theta(\eta)) = 0, \frac{d}{d\eta} v(\eta) + 1.380097266\,(f(\eta).v(\eta)) - 1.380097266\,u(\eta)^2 - 0.1380097266\,u(\eta) - 2.723285714\,\theta(\eta) = 0, \frac{d}{d\eta} f(\eta) = u(\eta), \frac{d}{d\eta}\theta(\eta) = w(\eta), \frac{d}{d\eta} u(\eta) = v(\eta) \right\}$$

$>IC := \{f(0) = -S, u(0) = 1, \theta(0) = 1, v(0) = \alpha, w(0) = \beta\};$

$$IC := \{f(0) = -0.1, \theta(0) = 1, u(0) = 1, v(0) = \alpha, w(0) = \beta\}$$

$>L := 1.5;$

$$L := 1.5$$

$>BC := \{u(L) = 0, \theta(L) = 0\};$

$$BC := \{\theta(1.5) = 0, u(1.5) = 0\}$$

$>infolevel[shoot] := 1 :$

$>S := shoot(ODE, IC, BC, FNS, [\alpha = -2.27284266, \beta = -1.0171923401906]) :$

shoot: Step # 1

shoot: Parameter values : alpha = -2.27284266 beta = -1.0171923401906

shoot: Step # 2

shoot: Parameter values : alpha = HFloat(-2.1088770166729986) beta = HFloat(-1.7050425213143763)

shoot: Step # 3

shoot: Parameter values : alpha = HFloat(-2.042182010242269) beta = HFloat(-1.812759387262806)

shoot: Step # 4

shoot: Parameter values : alpha = HFloat(-2.044007324831599) beta = HFloat(-1.8103737971583949)

shoot: Step # 5

shoot: Parameter values : alpha = HFloat(-2.044008916631473) beta = HFloat(-1.810371708620083)

>

> u11 := odeplot(S, [η, u(η)], 0 ..L, color = green, linestyle = dash, *labels* = ["η", "f '(η)"], *labeldirections* = [*horizontal*, *vertical*], *labelfont* = [*timesnewroman*, 12]) :

> θ11 := odeplot(S, [η, θ(η)], 0 ..L, color = green, linestyle = dash, *labels* = ["η", "θ(η)"], *labeldirections* = [*horizontal*, *vertical*], *labelfont* = [*timesnewroman*, 12]) :

$>Pr := 6.2;\ S := 0.1;\ \Upsilon := 0.2;\ p := 3;\ \lambda := 0.1;$

$$Pr := 6.2$$
$$S := 0.1$$
$$\Upsilon := 0.2$$
$$p := 3$$
$$\lambda := 0.1$$

$> (\rho s) := 8933;\ (cps) := 385;\ (ks) := 401;\ (\beta s) := 4.89E-5;\ (\rho f) := 997;\ (cpf) := 4179;\ (kf) := 0.613;\ (\beta f) := 63E-5;$

$$\rho s := 8933$$
$$cps := 385$$
$$ks := 401$$

$$\beta s := 0.0000489$$

$$\rho f := 997$$

$$cpf := 4179$$

$$kf := 0.613$$

$$\beta f := 0.00063$$

$>(Kfn) := (ks) + 2.(kf) - 2.\Upsilon.((kf) - (ks));$

$$Kfn := 562.3808$$

$>A := (1-\Upsilon)^{2.5}.\left(1-\Upsilon+\Upsilon.\frac{(\rho s)}{(\rho f)}\right);$

$$A := 1.483733599$$

$>C := \left(1-\Upsilon+\Upsilon.\frac{(\rho s).(cps)}{(\rho f).(cpf)}\right);$

$$C := 0.9650899096$$

$>\boldsymbol{E} := \left(\boldsymbol{1-\Upsilon+\Upsilon.\frac{(\beta s)}{(\beta f)}}\right);$

$$E := 0.8155238095$$

$>(Kf) := (ks) + 2.(kf) + 2.\Upsilon.((kf) - (ks));$

$$Kf := 242.0712$$

$>E := \frac{Kfn}{Kf};$

$$E := 2.323204082$$

$>$

$>FNS := \{f(\eta), u(\eta), v(\eta), \theta(\eta), w(\eta)\}:$

$>ODE := \Big\{\mathrm{diff}(f(\eta), \eta) = u(\eta), \mathrm{diff}(u(\eta), \eta) = v(\eta), \mathrm{diff}(\theta(\eta), \eta) = w(\eta), diff(v(\eta), \eta) + A.\big((f(\eta).v(\eta) - u(\eta)^2) - (\lambda).u(\eta)\big) - \mathrm{B.p.}\theta(\eta) = 0, \ \mathrm{diff}(w(\eta), \eta) + \left(\frac{Pr.(C)}{E}\right).(f(\eta).w(\eta) - u(\eta).\theta(\eta)) = 0\Big\};$

$$ODE := \Big\{\frac{\mathrm{d}}{\mathrm{d}\eta} w(\eta) + 2.575562554\,(f(\eta).w(\eta)) - 2.575562554\,(u(\eta).\theta(\eta)) = 0, \frac{\mathrm{d}}{\mathrm{d}\eta} v(\eta) + 1.483733599\,(f(\eta).v(\eta)) - 1.483733599\,u(\eta)^2 - 0.1483733599\,u(\eta) - 2.723285714\,\theta(\eta) = 0, \frac{\mathrm{d}}{\mathrm{d}\eta} f(\eta) = u(\eta), \frac{\mathrm{d}}{\mathrm{d}\eta}\theta(\eta) = w(\eta), \frac{\mathrm{d}}{\mathrm{d}\eta} u(\eta) = v(\eta)\Big\}$$

$>IC := \{f(0) = -S, u(0) = 1, \theta(0) = 1, v(0) = \alpha, w(0) = \beta\};$

$$IC := \{f(0) = -0.1, \theta(0) = 1, u(0) = 1, v(0) = \alpha, w(0) = \beta\}$$

$>L := 1.5;$

$$L := 1.5$$

$>BC := \{u(L) = 0, \theta(L) = 0\};$

$$BC := \{\theta(1.5) = 0, u(1.5) = 0\}$$

$>infolevel[shoot] := 1:$

>$S := shoot(ODE, IC, BC, FNS, [\alpha = -2.2951750121166, \beta = -0.89890571906])$:

shoot: Step # 1

shoot: Parameter values : alpha = -2.2951750121166 beta = -.89890571906

shoot: Step # 2

shoot: Parameter values : alpha = HFloat(-2.1972690835890853) beta = HFloat(-1.3714170527350527)

shoot: Step # 3

shoot: Parameter values : alpha = HFloat(-2.1769900926954584) beta = HFloat(-1.4056629897343726)

shoot: Step # 4

shoot: Parameter values : alpha = HFloat(-2.1771230246979005) beta = HFloat(-1.4055235092059257)

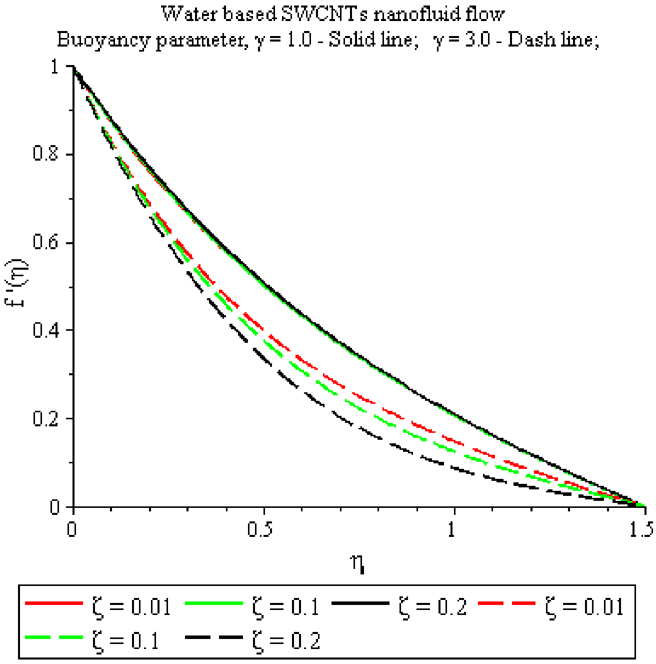
>

> u12 := odeplot(S, [η, u(η)], 0 ..L, color = black, linestyle = dash, *labels* = ["η", "f '(η)"], *labeldirections* = [*horizontal*, *vertical*], *labelfont* = [*timesnewroman*, 12]) :

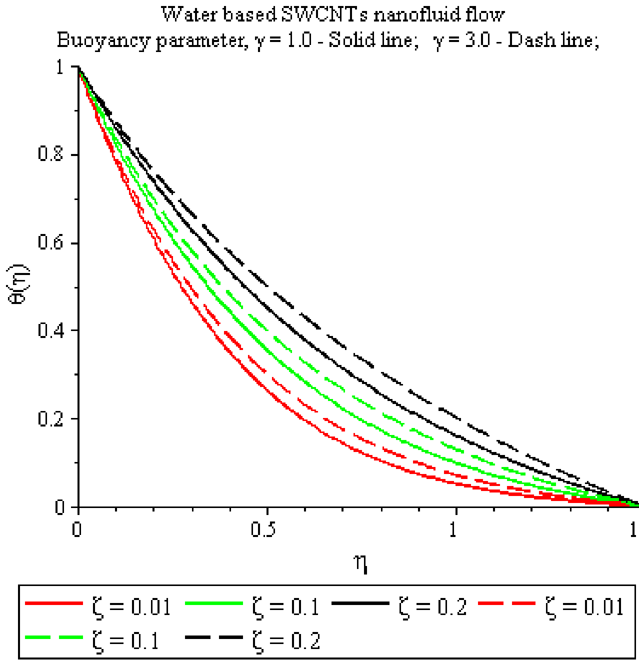
> θ12 := odeplot(S, [η, θ(η)], 0 ..L, color = black, linestyle = dash, *labels* = ["η", "θ(η)"], *labeldirections* = [*horizontal*, *vertical*], *labelfont* = [*timesnewroman*, 12]) :

display(u1, u2, u3, u4, u5, u6);

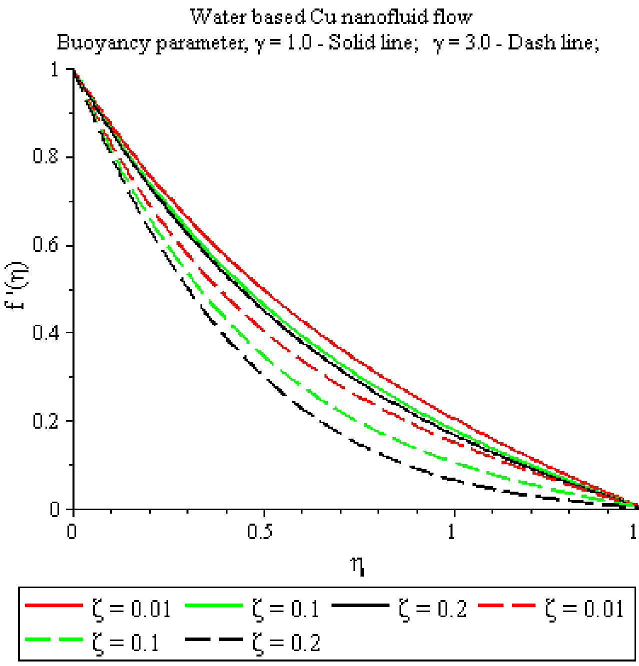
>*display*(*u1*, *u2*, *u3*, *u4*, *u5*, *u6*);



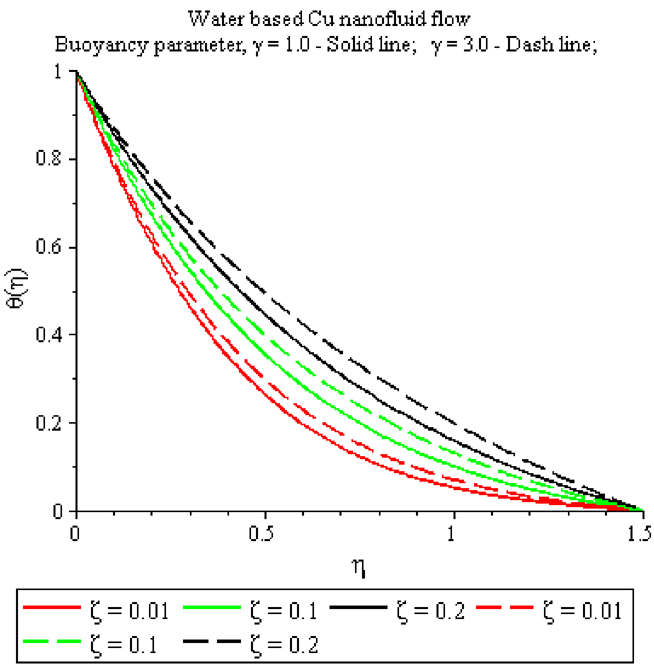
>*display*(*θ1*, *θ2*, *θ3*, *θ4*, *θ5*, *θ6*);



$>display(u7, u8, u9, u10, u11, u12);$



$>display(\theta 7, \theta 8, \theta 9, \theta 10, \theta 11, \theta 12);$


Water based Cu nanofluid flow
Buoyancy parameter, γ = 1.0 - Solid line; γ = 3.0 - Dash line;
θ(η)
η
ζ = 0.01
ζ = 0.1
ζ = 0.2
ζ = 0.01
ζ = 0.1
ζ = 0.2