



**WOLLEGA UNIVERSITY
SCHOOL OF GRAUATE STUDIES**

**LIGHT EMISSION FROM TWO-LEVEL ATOMS
INTERACTING WITH A SQUEEZED VACUUM**

MSc. THESIS

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NEKMTE,ETHIOPIA
January 12, 2026



**WOLLEGA UNIVERSITY
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**Light Emission from Two-Level Atoms Interacting with a
Squeezed Vacuum**

**A Thesis submitted to the College of Natural and Computational
Sciences in Partial Fulfillment of the Requirements for the Degree of
the Master of Science in Physics (Quantum Optics)**

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APPROVAL SHEET FOR SUBMITTING FINAL THESIS

As members of the Board of Examiners of the Final MSc. thesis open defense, we certify that we have read and evaluated the thesis prepared by **Rahel Tesfaye** under the title "**Light Emission from Two-Level Atoms Interacting with a Squeezed Vacuum**" and recommend that the thesis be accepted as fulfilling the thesis requirement for the **Degree of Master of Science in Physics (Quantum Optics)**

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Certification of the Final Thesis I hereby certify that all the corrections and recommendation suggested by the board of examiners are incorporated into the final thesis entitled "**Light Emission from Two-Level Atoms Interacting with a Squeezed Vacuum**" by **Rahel Tesfaye**.

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STATEMENT OF THE AUTHOR

I miss **Rahel Tesfaye** hereby and declare and affirm that the thesis entitled "**Light Emission from Two-Level Atoms Interacting with a Squeezed Vacuum**" is my work conducted under the supervision of **Misganu Chewaka (PhD)**. I have followed all the ethical principles of scholarships in all comments made during the proposal defense. Finally, the thesis was approved by the department graduate committee and prepared according to university criteria.

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BIOGRAPHICAL SKETCH

The author was born in Ilubabor Zone, Oromia, Ethiopia. She began her education at Alge Elementary School and completed her secondary education at Alge High School. She earned a Bachelor of Science (B.Sc.) degree in Applied Physics from Metu University. Subsequently, she joined Wollega University to pursue a Master of Science (M.Sc.) degree in Quantum Optics.

ACKNOWLEDGMENT

I begin by expressing my deepest gratitude to Almighty God, the ultimate source of my strength and inspiration, who has guided me to this significant juncture. My deepest appreciation also goes to my esteemed advisor, Dr. Misganu Chewaka (PhD), whose insightful and unwavering guidance was invaluable throughout the development of this thesis. Lastly, I would like to thank my parents and classmates for their immense patience and encouragement, which were essential to enabling me to complete this endeavor.

Acronyms and Abbreviations

EIT	Electromagnetically Induced Transparency
FFT	Fast Fourier Transform
H	Hamiltonian
JCM	Jaynes-Cummings Model
MATLAB	Matrix Laboratory
N_a	Number of Atoms
PSD	Power Spectral Density
QRT	Quantum Regression Theorem
SE	Spontaneous Emission
TLA	Two-Level Atom

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ABSTRACT

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0.1 Significance of the Study

This study provides a comprehensive theoretical and numerical investigation of light emission from a two-level atom interacting with a squeezed vacuum field. By developing a model based on the squeezed Jaynes–Cummings Hamiltonian and formulating atomic evolution via the master equation, the work establishes a framework for understanding how quantum squeezing modifies atomic dynamics. Analysis of population inversion and emission characteristics shows that squeezing can enhance or inhibit spontaneous emission, accelerate population oscillations, and induce spectral narrowing, while the second-order correlation function $g^{(2)}(\tau)$ reveals photon antibunching and nonclassical statistics. The study further examines two-atom systems coupled to a common squeezed vacuum, highlighting steady-state entanglement and nonclassical correlations in quadrature variances, emphasizing the role of squeezing as a quantum resource. Overall, these findings provide insight into the influence of squeezed reservoirs on spontaneous emission, coherence, photon correlations, and entanglement, with important implications for quantum optics, precision spectroscopy, quantum metrology, and quantum information processing.

Keywords: Master equation; Power spectrum; Quantum entanglement; Spontaneous emission; Squeezed vacuum; Two-level atom.

Chapter 1

INTRODUCTION

1.1 Background of the Study

The interaction between matter and the quantized electromagnetic field forms the foundation of modern quantum optics. For the simplest quantum emitter—a two-level atom—coupling to the vacuum field governs essential processes such as spontaneous emission, resonance fluorescence, atomic coherence, and quantum noise characteristics. In the standard formulation, the vacuum is treated as an ordinary (zero-point or thermal) state characterized by minimum uncertainty fluctuations. However, when the vacuum is replaced by a squeezed vacuum, a nonclassical state with reduced fluctuations in one quadrature and enhanced fluctuations in the conjugate quadrature, the atom–field interaction exhibits qualitatively new behavior. Such squeezed reservoirs can modify decay rates, shift spectral features, and generate quantum interference effects that are not observable in ordinary vacuum environments.

In quantum theory, light consists of photons (Wang and Agarwal, 2023), and its properties are described through quantum states expressed in terms of creation and annihilation operators. Quantum states of light include coherent states, number states, chaotic (thermal) states, and squeezed states (Teich and Saleh, 1989). Coherent states are particularly important in quantum optics because they represent the closest quantum analogue to classical electromagnetic fields and appear naturally in laser systems Fabre and Treps, 2020. A two-level atom, modeled with only a ground and excited energy state, provides a fundamental system for examining the exchange of light and matter, and the interaction with coherent, near-resonant fields is central to many quantum optical processes White, 2009.

The interaction between a two-level atom and any engineered electromagnetic reservoir—including a squeezed vacuum—is effectively described within the open-quantum-systems framework. Starting from a system–reservoir Hamiltonian, the reservoir degrees of freedom are traced out, yielding a master equation that governs the evolution of

the atomic density matrix. When the reservoir is a squeezed vacuum, the resulting Lindblad terms acquire additional phase-dependent contributions that influence both population decay and atomic coherence. These modifications make the phase and quadrature structure of the reservoir highly relevant to measurable quantities such as emission spectra and photon statistics. The master-equation and quantum Langevin formalisms thus provide powerful tools for predicting how squeezing influences atomic dynamics.

A squeezed vacuum is typically produced via nonlinear optical processes, such as parametric down-conversion, where quantum noise in one quadrature is reduced below the vacuum limit while the conjugate quadrature acquires enhanced noise Barsotti, Harms, and Schnabel, 2018. The degree and phase of squeezing are quantified by the squeezing parameter, which determines how the noise redistribution affects atomic emission and coherence Kassahun, 2012. When a two-level atom interacts with such a nonclassical reservoir, its emitted light acquires unique quantum features, including reduced noise in one quadrature and increased noise in the orthogonal one Kassahun, 2008. These noise asymmetries influence the structure of the emission spectrum and the temporal behavior of the atomic dipole.

Coupling a two-level atom to a squeezed reservoir gives rise to a wide range of physical phenomena. Depending on the squeezing strength and phase, spontaneous emission can be suppressed or enhanced, resulting in either subnatural linewidths or broadened spectral peaks. Squeezing can also induce transparency-like effects and generate asymmetric line shapes in fluorescence or absorption spectra due to interference between modified decay channels. Moreover, the nonclassical reservoir leaves signatures in higher-order correlations, producing modified second-order correlation functions $g^{(2)}(\tau)$ and altered squeezing spectra of the fluorescence. These theoretical predictions have been extensively explored in both free-space and cavity-QED environments.

The increasing ability to engineer quantum reservoirs has made these effects experimentally significant. In cavity-QED and circuit-QED platforms, squeezed states can be created using parametric amplifiers or nonlinear optical devices. This makes it possible to manipulate emission properties—such as decay rate, linewidth, and noise characteristics—without altering the emitter itself. Consequently, understanding the interaction between a two-level atom and a squeezed vacuum field bridges fundamental quantum optics with emerging technologies in precision metrology, noise-reduced spectroscopy, and quantum information processing.

Despite extensive theoretical progress, several open questions continue to motivate studies in this area. These include determining how different squeezing parameters (magnitude, phase, bandwidth) influence observables such as spectra and correlation functions under realistic decay channels; distinguishing cavity-enhanced squeezing effects from broadband free-space squeezing; and identifying parameter regimes where nonclassical signatures—such as sub-Poissonian statistics or subnatural linewidths—remain experimentally robust. A comprehensive analysis involving population dynamics, co-

herence evolution, correlation functions, and spectral calculations is therefore essential for further advancing understanding and guiding experimental applications.

In this work, we adopt a density-matrix master-equation approach to investigate the emission properties of a two-level atom interacting with a squeezed vacuum reservoir. Specifically, we derive the master equation including squeezing-dependent dissipative terms, analyze the evolution of populations and coherences, compute fluorescence spectra and second-order correlation functions, and examine how the squeezing parameters—strength, phase, and bandwidth—govern enhancement, suppression, and interference effects in atomic emission. These results provide valuable insights into how engineered squeezed reservoirs can be used to tailor light emission for fundamental and technological applications.

1.2 Statement of the Problem

Squeezed vacuum fields play a central role in modern quantum optics due to their ability to modify fundamental light–matter interaction processes; however, a detailed understanding of how such nonclassical fields influence light emission from two-level atomic systems remains incomplete. The phase-dependent quantum fluctuations associated with squeezed reservoirs introduce noise asymmetries that significantly alter spontaneous emission rates, coherence dynamics, spectral properties, and photon statistics, effects that cannot be adequately described using simple analytical models. A rigorous theoretical treatment therefore requires a density-matrix master-equation formalism that explicitly incorporates squeezing-dependent terms, whose solutions become increasingly complex under realistic conditions such as strong squeezing, detuning, decoherence, and multi-atom interactions. Experimental investigation of these phenomena is further constrained by the difficulty of generating stable squeezed fields and achieving precise control over atom–field coupling. Consequently, numerical approaches, particularly MATLAB-based simulations, provide an essential framework for systematically analyzing atomic population dynamics, emission spectra, entanglement behavior, and second-order correlation functions $g^{(2)}(\tau)$. Developing such a theoretical and numerical framework is therefore necessary to clarify the role of squeezed vacuum fields in shaping atomic emission processes and to guide future applications in quantum information and communication technologies.

1.3 General Objective of the Study

The general objective of this study is to develop a theoretical and numerical framework for analyzing light emission from two-level atoms coupled to a squeezed vacuum reservoir.

1.3.1 Specific Objectives of the Study

The specific objectives of this study are:

- To develop a mathematical model describing the interaction of a two-level atom with a squeezed vacuum field using the squeezed Jaynes–Cummings Hamiltonian.
- To analyze atomic dynamics by solving the master equation that incorporates the effects of a phase-sensitive squeezed reservoir.
- To examine the time evolution of atomic population inversion under different squeezing parameters through numerical simulations.
- To determine the power spectrum, second-order correlation function, two-atom entanglement, and quadrature variances to reveal nonclassical correlations.

1.4 Significance of the Study

The significance of this study lies in its comprehensive theoretical and numerical investigation of light emission from a two-level atom interacting with a squeezed vacuum field, revealing how quantum squeezing fundamentally modifies atomic dynamics and emission properties. By employing the squeezed Jaynes–Cummings Hamiltonian, master equation, and quantum regression theorem, the study demonstrates that squeezing enhances the effective Rabi frequency, accelerates population oscillations, and alters the emission spectrum in a phase-sensitive manner. The results show subnatural linewidth narrowing and quantum noise suppression, key signatures of nonclassical light–matter interaction. Furthermore, the analysis of photon correlations and quadrature variances confirms strong two-photon coherence and antibunching behavior, while simulations of two atoms in a common squeezed reservoir reveal steady-state entanglement generation, emphasizing the role of squeezing as a quantum resource. Overall, the findings provide deeper insight into how squeezed vacuum fields influence spontaneous emission, coherence, and entanglement, offering valuable implications for applications in quantum optics, precision spectroscopy, quantum metrology, and quantum information processing.

Chapter 2

REVIEW OF LITERATURE

2.1 Squeezed States of Light

The generation of light fields that are intrinsically quantum mechanical has been a topic of interest in recent years. In particular, squeezed states of light are now well understood theoretically and have been generated in several nonlinear optical systems (Qin et al., 2024).

The interaction of two-level atoms with squeezed light has been extensively studied, as it exhibits interesting non-classical features. This kind of interaction is involved in various physical processes of interest, such as resonance fluorescence and laser dynamics. The width of the incoherent spectrum of the fluorescent light has been modified by the squeezed light (Abdel-Wahab, Ibrahim, and Amin, 2024).

A two-level laser is a source of coherent or chaotic light emitted by two-level atoms inside a cavity coupled to a vacuum reservoir via a single-port mirror (Kassahun, 2012). In one model of such a laser, two-level atoms initially in the upper level are injected at a constant rate into a cavity and removed after they have decayed due to spontaneous emission (Gardiner, 1986). The interaction between the squeezed vacuum state and the two-level atom can be described by the (Fakhri, Mirzaei, and Sayyah-Fard, 2021) Jaynes-Cummings Hamiltonian: The system We are considering a system where a squeezed vacuum state interacts with a two-level atom. The squeezed vacuum state is a quantum state of light with reduced quantum noise in one quadrature and increased noise in the orthogonal quadrature (Gerry and Welch, 1991).

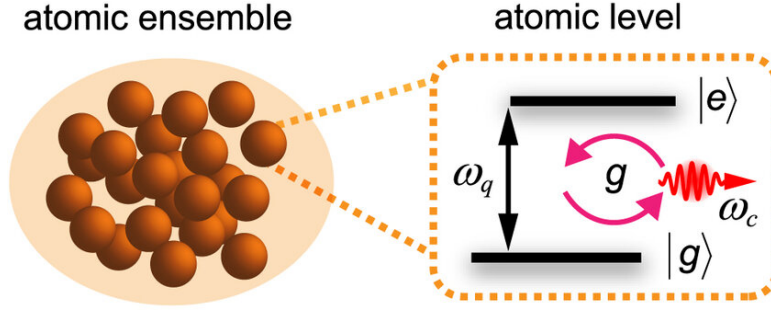


Figure 2.1: Schematic diagram of two-level atoms with the ground state $|g\rangle$ and excited state $|e\rangle$, interacting with the quantized cavity field of frequency ω_c .

2.2 The interaction of two-level atoms with a squeezed vacuum

2.2.1 Spontaneous emission rates

Spontaneous emission (SE) is the process by which an excited atom emits a photon and decays to a lower energy (Friedman et al., 1988) state without any external stimulation. The rate of this process is typically calculated in the context of an atom interacting with the ordinary electromagnetic vacuum. However, when the atom is immersed in a squeezed vacuum, the SE rate can be significantly modified. A squeezed vacuum state is a nonclassical state of light where the quantum fluctuations of one of the field quadratures are reduced below the standard quantum limit, at the expense of increased fluctuations in the conjugate quadrature. This is different from the ordinary vacuum, where fluctuations are equal in both quadratures. The spontaneous emission rate can be either enhanced or suppressed depending on the relative phase between the atomic dipole and the squeezing phase of the vacuum. If the phases are aligned constructively, the SE (Abrantes, Szilard, and Farina, 2025) rate is enhanced; if they are aligned destructively, it is suppressed. This is a key departure from the standard SE process, which has (Y. Zhang et al., 2022) a fixed rate. The dependence on the squeezing phase is a unique and important feature. The squeezing phase can be controlled experimentally, allowing for the manipulation of the spontaneous emission rate. This provides a mechanism for quantum control over atomic decay.

2.2.2 Suppressed atomic coherency

Suppressed atomic coherency refers to the reduction of the coherence (Japha et al., 2016) of an atomic system. In quantum mechanics, coherence describes the existence of a fixed phase relationship between different quantum states, allowing for quantum

superposition and interference. When this phase relationship is lost, the system is said to have decohered. **Spontaneous Emission:** In an ordinary vacuum, the fundamental limit to atomic coherence is the spontaneous emission of photons. When an excited atom decays, the emission of a photon carries (Pollnau, 2018) away information about the atomic state, destroying the phase relationship between the excited and ground states. This is a primary source of decoherence for atoms.

2.2.3 Steady-state entanglement

Steady-state entanglement refers to the ability to (Horodecki et al., 2009) create and maintain quantum entanglement between two or more systems for a prolonged, and in principle, indefinite period. This is a significant challenge because quantum systems are inherently prone to decoherence, which is the loss of quantum properties, including entanglement, due to interactions with their environment. **Dissipative Entanglement:** A shared dissipative channel is used to create and maintain entanglement. For example, two atoms interacting with the same electromagnetic vacuum can be driven into a collective entangled state. The dissipation, rather than destroying the entanglement, becomes the mechanism that generates and stabilizes it. **Driven-Dissipative Systems:** A constant external drive, such as a laser, is used to continuously pump energy into the system. This energy balances the dissipation, creating a stable, entangled state. This method is often used in cavity quantum electrodynamics, where atoms are coupled to a driven optical cavity.

2.2.4 squeezing-induced transparency in the emission spectrum

Squeezing-induced transparency in the emission spectrum is a quantum interference effect that makes an otherwise opaque atomic medium transparent to light at a specific frequency. This phenomenon is analogous to electromagnetically induced transparency (EIT) but uses a squeezed vacuum field instead of a strong coherent laser field to create the transparency window (Ficek and Dalton, 1993). In a typical atomic system, a two-level atom absorbs and re-emits photons at its resonance frequency, making the medium opaque. The absorption is due to transitions from the ground state to an excited state. The atom then decays back to the ground state via spontaneous emission (Lvovsky, 2015) which broadens the absorption line. When the atom is placed in a squeezed vacuum, the situation changes dramatically (Carmichael, 1992). A squeezed vacuum is a non-classical state of light where the quantum fluctuations of one of the field quadratures are reduced below the vacuum noise level (Fox, 2006).

2.3 Atom field interaction

Atom-field coupling strength quantifies the intensity of the interaction between an atom and an electromagnetic field. This (Abdel-Wahab, Ibrahim, and Amin, 2024) dictates how strongly the presence of the field influences the atom's internal states. A stronger coupling means a more significant exchange of energy and information between the atom and the field.

The coupling strength is a numerical value, often expressed in frequency units, which represents the rate of energy exchange between the atom and the field (Judd, 2014). A Hamiltonian equation of the system of its two-level atom, the squeezed vacuum field, and their interaction is given as

$$H = \hbar\omega_a\sigma_+\sigma_- + \hbar\omega_c a^\dagger a + \hbar g(a\sigma_+ + a^\dagger\sigma_-) \quad (2.1)$$

where: - ω_a is the atomic transition frequency, - ω_c is the cavity mode frequency, - g is the coupling constant, - a and $a^\dagger$ are the annihilation and creation operators for the cavity mode, - σ_+ and σ_- are the raising and lowering operators for the atom.

$|\xi\rangle$ is defined by the squeezing parameter $\xi = re^{i\theta}$, where r is the squeezing magnitude and θ is the squeezing angle. The state can be written as:

$$(|\xi\rangle = S(\xi)|0\rangle) \quad (2.2)$$

where $S(\xi)$ is the squeezing operator. The time evolution of system can be found by solving the Schrodinger equation with the Hamiltonian H . The state of the system at time t is:

$$|\psi(t)\rangle = e^{-\frac{iHt}{\hbar}} |\psi(0)\rangle \quad (2.3)$$

where $|\psi(0)\rangle$ is the initial state, typically the atom in the ground state and the cavity in the squeezed vacuum state. The light emission from the system can be analyzed by considering the expectation values of the photon number operator $a^\dagger a$ and the quadrature operators X and Y . The squeezed vacuum state will affect the noise properties of these operators, leading to non-classical light emission. The light emission from the squeezed vacuum high interaction with a two-level atom is characterized by non-classical noise properties, with reduced noise in one quadrature and increased noise in the orthogonal quadrature. The Hamiltonian interaction and the evolution of time describe the dynamics of the system, leading to specific emission patterns influenced by the squeezing parameters. Squeezed states are usually not characterized by counting their photons but by measuring canonical continuous-variable phase-space observable. Measurements are performed, as usual, on an ensemble of identical states, and quasi-probability density functions are calculated from the data (Schnabel, 2017).

The dynamics of the system can be described by the master equation for the density matrix ρ :

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[H_I, \rho] + \mathcal{L}(\rho) \quad (2.4)$$

where $\mathcal{L}(\rho)$ represents the Lindblad terms accounting for dissipation and decoherence. The vacuum squeezed state $|\xi\rangle$ is defined by the squeezing parameter $\xi = re^{i\theta}$, where r is the magnitude of squeezing and θ is the angle of squeezing. The squeezed vacuum state can be written as:

$$|xi\rangle = S(\xi)|0\rangle \quad (2.5)$$

where $S(\xi)$ is the squeezing operator:

$$S(\xi) = \exp\left(\frac{1}{2}(\xi^* a^2 - \xi (a^\dagger)^2)\right) \quad (2.6)$$

To simplify the dynamics, we move to the interaction picture. The interaction Hamiltonian in the interaction picture is:

$$H_I = \hbar g(ae^{-i\omega_0 t}|e\rangle\langle g| + a^\dagger e^{i\omega_0 t}|g\rangle\langle e|) \quad (2.7)$$

The spontaneous-emission spectrum of a two-level atom damped into an ordinary vacuum is a Lorentzian function of frequency with the width given by the atomic linewidth. A standard procedure to calculate the squeezing effects is to derive an appropriate master equation (Boyanovsky, 2015) describing the atomic evolution, which together with the quantum regression theorem is then used to calculate the spectrum. The derivation of the master equation is based on the assumption of the Born and Markov approximations, the latter requires a broadband spectrum of the reservoir modes (Messikh, Tanaś, and Ficek, 2000)

2.4 Power spectrum

The power spectrum of the light scattered by a two-level atom in the presence of a coherent continuous-pulse-train driving field is analyzed. Separate expressions are obtained for the coherent and incoherent components of the spectrum. The coherent part of the spectrum consists of a series of spikes (Newbold and Salamo, 1980) which are displaced from the optical carrier by multiples of the pulse repetition frequency. The incoherent spectrum can have fixed peaks at these same frequencies plus sidebands equally spaced on opposite sides of these fixed peaks. Incident light interacts with the system, causing transitions between levels. We are interested in the spectrum of the scattered light. The power spectrum is the Fourier transform of the autocorrelation function of the scattered electric field. This involves calculating the correlation function and then performing the Fourier transform to obtain the spectral density. This will reveal the frequencies present in the scattered light with peaks at frequencies related to ω_0 and the incident light frequency. The power spectrum is estimated using Fourier transform methods such as the

(Same et al., 2020) Welch method and the filter bank method. For signals with relatively small length, the filter bank approach produces a spectral estimate with a higher resolution, a more accurate noise floor, and peaks more precise than the Welch method, with low or no spectral leakage. Power Spectral Density (PSD) is an essential representation of the signal spectrum that depicts the power measurement content versus frequency. Power Spectral Density is typically used to characterize broadband random signals and has a variety of usages in many fields like physics, engineering, biomedical, etc. This paper proposes a simple and practical method to estimate the PSD based on the Welch algorithm for spectrum monitoring. The Jaynes-Cummings model (JCM) is a fundamental model in quantum optics that describes the resonant interaction between a two-level atom and a single mode of a quantized electromagnetic field. The Hamiltonian (Scully and Zubairy, 1997) for the standard JCM is given by:

$$(H_{JCM} = \hbar\omega_c a^\dagger a + \frac{1}{2}\hbar\omega_a \sigma_z + \hbar g(a^\dagger \sigma_- + a \sigma_+)) \quad (2.8)$$

where:

- $\hbar\omega_c$ is the energy of the cavity mode, with $a^\dagger$ and a being the creation and annihilation operators for photons.
- $\frac{1}{2}\hbar\omega_a \sigma_z$ is the energy of the two-level atom, with σ_z being the Pauli z-matrix.
- $\hbar g(a^\dagger \sigma_- + a \sigma_+)$ describes the atom-field interaction, with g being the coupling constant, and σ_- and σ_+ being the atomic lowering and raising operators, respectively.

When the JCM is “modified to include interaction with a squeezed vacuum state,” it means that the electromagnetic field mode interacting with the atom is not in a simple vacuum or coherent state, but rather in a **squeezed vacuum state**. A squeezed vacuum state is a non-classical state of (Walls and Milburn, 2008) light characterized by reduced quantum noise in one quadrature phase at the expense of increased noise in the orthogonal quadrature. This reduction in noise is a direct consequence of non-zero correlations between the field quadratures. The “non-zero correlations” mentioned refer to the fact that for a squeezed vacuum state, the expectation value of products of annihilation and creation operators do not follow the simple rules of an unsqueezed vacuum. Specifically, for a squeezed vacuum, $\langle aa \rangle \neq 0$ and $\langle a^\dagger a^\dagger \rangle \neq 0$, even though $\langle a \rangle = 0$ and $\langle a^\dagger \rangle = 0$. These non-zero correlations are what distinguish squeezed states from coherent states or thermal states and lead to their unique noise properties. The specific form of the Hamiltonian including (Walls and Milburn, 2008) interaction with a squeezed vacuum state can vary depending on how the squeezing is introduced (through a squeezed bath, or by directly preparing the field in a squeezed state). However, it generally involves terms that reflect these non-zero correlations. For instance, terms

proportional to a^2 or $(a^\dagger)^2$ might appear, which are responsible for generating or interacting with squeezed light. This Hamiltonian describes a process where the field energy is exchanged with a “squeezing reservoir” or a strong classical pump (in the case of parametric amplification) that prefers to create or destroy photons in pairs. The presence of the a^2 and $(a^\dagger)^2$ terms is the hallmark of squeezing. When this Hamiltonian acts on the vacuum state $|0\rangle$, it generates a **squeezed vacuum state**. This state has zero mean photon number $\langle a^\dagger a \rangle = 0$, but its quantum fluctuations in the field quadratures are highly anisotropic. One quadrature will have reduced noise below the “standard quantum limit” (the vacuum noise level), while the conjugate quadrature will have enhanced noise, ensuring the Heisenberg uncertainty principle is still obeyed. In summary, the Squeezed Vacuum Field Hamiltonian mathematically describes the energy and the non-classical two-photon processes that lead to the generation or interaction of a quantized electromagnetic field with a squeezed vacuum state.

The Squeezed Vacuum (Gietka, 2023) Field Hamiltonian describes the dynamics of an electromagnetic field mode that is in a squeezed vacuum state. Unlike a regular vacuum state, which has equal quantum fluctuations in its two conjugate quadratures (like position and momentum for a mechanical oscillator, or electric and magnetic field amplitudes for light), a squeezed vacuum state has reduced fluctuations in one quadrature at the expense of increased fluctuations in the other, while still maintaining the minimum uncertainty product allowed by the Heisenberg Uncertainty Principle.

The Hamiltonian that governs such a squeezed field typically contains terms that create or annihilate photons in pairs. This is because squeezing is often generated through nonlinear optical processes, such as parametric amplification, where pump photons are converted into pairs of signal and idler photons (Lvovsky, 2015). To describe a squeezed vacuum, an additional term is added to the Hamiltonian that accounts for the squeezing interaction. This interaction term is typically quadratic in the field operators, representing the creation or annihilation of photon pairs. The most common form of the Squeezed Vacuum Field Hamiltonian (or more precisely, the Hamiltonian that generates squeezing from the vacuum, often called the Squeezing Hamiltonian or Parametric Amplifier Hamiltonian) is:

$$H_{Sq} = \hbar\omega a^\dagger a + \hbar \left[\frac{\kappa}{2} (a^2 e^{-i\phi} + (a^\dagger)^2 e^{i\phi}) \right] \quad (2.9)$$

2.5 Quantum collapses and revivals

Collapse and revival occur for many phenomena in quantum physics appearing in different aspects and forms (Mährlein et al., 2020). Typically, an observable of a system falls off in time to vanish at a given moment only to reappear again, sometimes even to its original value, sometimes periodically. A prominent example from atomic physics is a two-level atom interacting with a single mode of a cavity, as described by the Jaynes-

Cummings model. Three identical two-level atoms in free space prepared in particular entangled single-photon excited states display a “birth,” “death,” and a revival of spontaneous emission in selected directions.

2.6 Evolution of the Density Matrix

The evolution of the total system density matrix ρ_{total} is (Y. Chen and A.-X. Chen, 2025) given by the Liouville-von Neumann equation.

For a quantum system that is isolated and evolves unitarily, the time evolution of its density matrix $\rho(t)$ is governed by the **Liouville-von Neumann equation**:

$$i\hbar \frac{d\rho}{dt} = [H, \rho] \quad (2.10)$$

where:

- $\hbar$ is the reduced Planck constant.
- ρ is the density operator (or density matrix) of the system.
- H is the Hamiltonian operator of the system.
- $[H, \rho] = H\rho - \rho H$ is the commutator of the Hamiltonian and the density operator.

This equation is the quantum mechanical analogue of the classical Liouville equation for the probability distribution in phase space. It describes the coherent, unitary evolution of the system.

2.6.1 Solution for a time-independent Hamiltonian

If the Hamiltonian H is time-independent, the formal solution to the Liouville-von Neumann equation is:

$$\rho(t) = e^{-iHt/\hbar} \rho(0) e^{iHt/\hbar} \quad (2.11)$$

where $\rho(0)$ is the initial density matrix at time $t = 0$, and $e^{-iHt/\hbar}$ is the unitary time-evolution operator.

2.7 The Lindblad Master Equation

When a quantum system interacts with (Abebe et al., 2021) an environment, it is no longer closed, and its evolution is generally non-unitary. This interaction leads to

phenomena like decoherence and dissipation. The most general form of a Markovian (memoryless) master equation that preserves the physical properties of the density matrix (Hermiticity, trace, and complete positivity) is the **Lindblad master equation**, also known as the (Manzano, 2020) Gorini-Kossakowski-Sudarshan-Lindblad (GKSL) equation:

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[H, \rho] + \mathcal{L}_D(\rho) \quad (2.12)$$

where:

- The first term, $-\frac{i}{\hbar}[H, \rho]$, represents the coherent (unitary) evolution due to the system's Hamiltonian H , as in the Liouville-von Neumann equation.
- $\mathcal{L}_D(\rho)$ is the dissipative or Lindblad superoperator, which describes the interaction with the environment and is responsible for non-unitary dynamics. Its full form is:

$$\mathcal{L}_D(\rho) = \sum_k \left(L_k \rho L_k^\dagger - \frac{1}{2} \{L_k^\dagger L_k, \rho\} \right) \quad (2.13)$$

In this term:

- L_k are the **Lindblad operators** (also called jump operators or collapse operators). These operators (**watanabe2025universal**) describe the specific ways in which the system interacts with the environment (e.g., photon emission, dephasing, energy relaxation).
- $L_k^\dagger$ are their Hermitian conjugates.
- $\{A, B\} = AB + BA$ denotes the anticommutator.
- The summation is over all possible dissipative processes.

The evolution of the reduced density matrix ρ (after Born-Markov and tracing out the reservoir) leads to an integral form:

$$\frac{d\rho}{dt} = -\frac{1}{\hbar^2} \int_0^\infty dt' \text{Tr}_R ([H_{int,I}(t), [H_{int,I}(t-t'), \rho(t) \rho_R]]) \quad (2.14)$$

where $H_{int,I}(t) = \hbar \sum_k g_k (\sigma_+(t) b_k(t) - \sigma_-(t) b_k^\dagger(t))$.

2.7.1 Squeezed vacuum correlation functions

For a broadband squeezed vacuum, the non-zero correlation functions (Gerry and Knight, 2023) are

$$\langle b(t)b^\dagger(t') \rangle = N + 1 \langle b^\dagger(t)b(t') \rangle = N \langle b(t)b(t') \rangle = M e^{-i\phi} \langle b^\dagger(t)b^\dagger(t') \rangle = M^* e^{i\phi} \quad (2.15)$$

where $N = \sinh^2 r$ is the mean photon number, $M = \cosh r \sinh r$ is the squeezing parameter, and ϕ is the phase of squeezing.

2.7.2 The master equation (Lindblad Form)

The master equation for the (Gardiner, 1986) reduced atomic density matrix ρ , incorporating the squeezing parameters (N and M) in the Lindblad form, is:

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[H_{eff}, \rho] + \mathcal{L}_{diss}(\rho) \quad (2.16)$$

The evolution of the density matrix ρ for a two-level atom interacting with a squeezed vacuum reservoir is described by a master equation in the Lindblad form. In the interaction picture, and under the Born-Markov and rotating wave approximations, the master equation is given by:

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[H_{eff}, \rho] + \mathcal{L}_{sq}[\rho] \quad (2.17)$$

The effective Hamiltonian H_{eff} includes the atomic transition frequency ω_0 :

$$H_{eff} = \frac{\hbar\omega_0}{2}\sigma_z \quad (2.18)$$

The general compact form of the Lindblad master equation for the time evolution of the density matrix ρ of an open quantum system is (Shibata and Katsura, 2020) given by:

$$\frac{d\rho}{dt} = \mathcal{L}(\rho) = -\frac{i}{\hbar}[H, \rho] + \sum_k \left(L_k \rho L_k^\dagger - \frac{1}{2} \{ L_k^\dagger L_k, \rho \} \right) \quad (2.19)$$

Chapter 3

METHODOLOGY

3.1 Theoretical model

In this study, we considered two identical two-level atoms interacting with a broadband squeezed vacuum reservoir. This section explains how the modified theoretical aspects of light emission from squeezed vacuum light interact with two two-level atoms. The primary application of the Fourier transform is to obtain the frequency components of an input signal. For a discrete signal, the FFT can be applied to a certain number of samples or, depending on the sampling frequency, a certain amount of time of the signal. As stated by (Same et al., 2020) spectrum in DFT is limited to half of the sampling frequency.

3.2 Mathematical modeling

A two-level atom with excited state $|e\rangle$ and ground state $|g\rangle$, separated by energy $\hbar\omega_0$, interacting with a quantized radiation field prepared in a squeezed vacuum state. The environment modifies the spontaneous emission characteristics of the atom as a result of quantum correlations in the squeezed field. We focused a two-level atom with ground state $|g\rangle$, excited state $|e\rangle$, and transition frequency ω_0 . The atomic operators are

$$\sigma_- = |g\rangle\langle e|, \quad \sigma_+ = |e\rangle\langle g|, \quad \sigma_z = |e\rangle\langle e| - |g\rangle\langle g|. \quad (3.1)$$

The atom interacts with a broadband squeezed vacuum reservoir characterized by the squeezing parameter r and phase ϕ . The mean photon number and two-photon correlations of the field are

$$N = \sinh^2 r, \quad M = \sinh r \cosh r e^{i\phi}, \quad |M| \leq \sqrt{N(N+1)}. \quad (3.2)$$

3.3 Hamiltonian of the System

The total Hamiltonian for the system was formulated, comprising the free Hamiltonian of the two-level atom, the Hamiltonian of the squeezed vacuum field, and the interaction Hamiltonian describing the coupling between the atom and the field. The total Hamiltonian H for a system describing light emission from the interaction of a two-level atom with a squeezed vacuum light field can be expressed as a sum of three main parts: the atomic Hamiltonian (H_A), the field Hamiltonian (H_F), and the atom-field interaction Hamiltonian (H_{AF}).

$$H = H_A + H_F + H_{AF} \quad (3.3)$$

3.3.1 Atomic Hamiltonian (H_A)

This term describes the internal energy of the two-level atom.

$$H_A = \frac{1}{2}\hbar\omega_a\sigma_z \quad (3.4)$$

where:

- $\hbar$ is the reduced Planck constant.
- ω_a is the transition frequency of the two-level atom.
- σ_z is the Pauli Z-matrix, representing the energy difference between the excited and ground states of the atom.

3.3.2 Field Hamiltonian (H_F)

This term describes the energy of the quantized electromagnetic field. For a squeezed vacuum, beyond the usual free field energy, terms representing the squeezing interaction are included.

$$H_F = \hbar\omega_c a^\dagger a + \hbar \left[\frac{\kappa}{2} (a^2 e^{-i\phi} + (a^\dagger)^2 e^{i\phi}) \right] \quad (3.5)$$

where:

- ω_c is the frequency of the cavity mode (or the mode of the electromagnetic field).
- $a^\dagger$ and a are the creation and annihilation operators for photons in the mode. The term $\hbar\omega_c a^\dagger a$ represents the energy of the free electromagnetic field.
- κ is the squeezing parameter (or effective squeezing strength), which quantifies the degree of squeezing.

- ϕ is the squeezing angle (or phase of the squeezing).
- The terms $\frac{\kappa}{2}(a^2e^{-i\phi} + (a^\dagger)^2e^{i\phi})$ are crucial for describing the squeezed vacuum. These terms create or annihilate pairs of photons, which is characteristic of squeezed states.

3.3.3 Atom-field interaction Hamiltonian (H_{AF})

This term describes the interaction between the two-level atom and the electromagnetic field. It typically takes the form of the Jaynes-Cummings interaction, assuming a dipole coupling.

$$H_{AF} = \hbar g(a^\dagger \sigma_- + a \sigma_+) \quad (3.6)$$

where:

- g is the coupling constant between the atom and the field mode, representing the strength of their interaction.
- σ_- is the atomic lowering operator (transition from excited to ground state).
- σ_+ is the atomic raising operator (transition from ground to excited state).

Combining these components, the total System Hamiltonian for light emission from squeezed vacuum light interaction with a two-level atom is:

$$H = \frac{1}{2}\hbar\omega_a\sigma_z + \hbar\omega_c a^\dagger a + \hbar \left[\frac{\kappa}{2}(a^2e^{-i\phi} + (a^\dagger)^2e^{i\phi}) \right] + \hbar g(a^\dagger \sigma_- + a \sigma_+) \quad (3.7)$$

This Hamiltonian can be used to study various phenomena such as quantum statistics of emitted light, entanglement generation, and population dynamics of the atom in the presence of a squeezed vacuum environment.

3.4 Two-Level Atom Hamiltonian

The Two-Level Atom Hamiltonian is a fundamental concept in quantum mechanics and quantum optics used to describe the simplest possible quantum system that can interact with an electromagnetic field: an atom with only two relevant energy states. This Hamiltonian model an atom (or any quantum system like a qubit) as having effectively only two distinct energy levels. These are typically referred to as the ground state and the excited state. All other energy levels are assumed to be far off resonance or irrelevant to the particular interaction being studied, allowing for a significant simplification of the complex energy structure of the atom. The Hamiltonian describes the internal energy of this two-level system. It is commonly expressed as:

$$H_A = \frac{1}{2}\hbar\omega_0\sigma_z \quad (3.8)$$

where:

- H_A is the Hamiltonian for the atom.
- $\hbar$ is the reduced Planck constant.
- ω_0 (or ω_a as used previously) is the **transition frequency** between the two levels. This is related to the energy difference ΔE between the excited and ground states by

$$\Delta E = \hbar\omega_0 \quad (3.9)$$

- σ_z is the **Pauli Z-matrix**, a 2×2 matrix defined as:

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad (3.10)$$

In the basis where the excited state is represented by $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and the ground state by $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$, applying σ_z to these states yields:

$$\begin{aligned} - \sigma_z \begin{pmatrix} 1 \\ 0 \end{pmatrix} &= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ (excited state has an energy contribution of } +\frac{1}{2}\hbar\omega_0) \\ - \sigma_z \begin{pmatrix} 0 \\ 1 \end{pmatrix} &= \begin{pmatrix} 0 \\ -1 \end{pmatrix} \text{ (ground state has an energy contribution of } -\frac{1}{2}\hbar\omega_0) \end{aligned}$$

This mathematical form conveniently places the "zero" of energy midway between the two levels. This choice simplifies calculations, particularly when considering interactions with light. In essence, the Two-Level Atom Hamiltonian quantifies the intrinsic energy of the atom when it's either in its lower or higher energy state, providing the foundation for analyzing how it absorbs or emits photons. It's a cornerstone for understanding phenomena like atomic absorption, stimulated emission, spontaneous emission, and the operation of lasers and quantum computing qubits.

The total Hamiltonian of the system is composed of three parts:

$$H = H_{\text{atom}} + H_{\text{field}} + H_{\text{int}}, \quad (3.11)$$

where: - $H_{\text{atom}} = \frac{1}{2}\hbar\omega_0\sigma_z$ is the atomic Hamiltonian, - $H_{\text{field}} = \sum_k \hbar\omega_k a_k^\dagger a_k$ is the free Hamiltonian of the field, - $H_{\text{int}} = -\vec{d} \cdot \vec{E}$ is the interaction Hamiltonian under electric

dipole approximation. Using the rotating-wave approximation (RWA), the interaction Hamiltonian becomes:

$$H_{\text{int}} = \hbar \sum_k \left(g_k a_k \sigma_+ + g_k^* a_k^\dagger \sigma_- \right) \quad (3.12)$$

where: - $\sigma_+ = |e\rangle \langle g|$ and $\sigma_- = |g\rangle \langle e|$ are the atomic raising and lowering operators, - g_k is the coupling constant, - a_k and $a_k^\dagger$ are the annihilation and creation operators of the field mode k .

3.5 Two-Level Atom Coupled to a Squeezed Vacuum Field

The interaction between a two-level atom and a single quantized radiation mode in the interaction picture can be described by the Jaynes–Cummings Hamiltonian. When the field is in a *squeezed vacuum state*, one uses the Bogoliubov transformed annihilation operator

$$b = S(\zeta) a S^\dagger(\zeta) = \mu a + \nu e^{i\theta} a^\dagger, \quad (3.13)$$

where

$$\mu = \cosh r, \quad \nu = \sinh r, \quad \zeta = r e^{i\theta}. \quad (3.14)$$

Here, a and $a^\dagger$ are the usual photon annihilation and creation operators, r is the squeezing parameter, and θ is the squeezing phase. In the rotating-wave approximation (RWA), the interaction-picture Hamiltonian is

$$H_I(t) = \hbar g \left(\sigma_+ b e^{i\Delta t} + \sigma_- b^\dagger e^{-i\Delta t} \right), \quad (3.15)$$

where $\sigma_\pm$ are the atomic raising and lowering operators, g is the atom–field coupling constant, and $\Delta = \omega_0 - \omega_c$ is the atom–field detuning. Expanding b and $b^\dagger$, we obtain

$$H_I(t) = \hbar g \left[\mu \left(\sigma_+ a e^{i\Delta t} + \sigma_- a^\dagger e^{-i\Delta t} \right) + \nu e^{i\theta} \sigma_+ a^\dagger e^{i\Delta t} + \nu e^{-i\theta} \sigma_- a e^{-i\Delta t} \right]. \quad (3.16)$$

On resonance ($\Delta = 0$), the Hamiltonian reduces to

$$H_I = \hbar g \left[\mu \left(\sigma_+ a + \sigma_- a^\dagger \right) + \nu e^{i\theta} \sigma_+ a^\dagger + \nu e^{-i\theta} \sigma_- a \right]. \quad (3.17)$$

For $r = 0$ (i.e. $\mu = 1$, $\nu = 0$), this reduces to the standard Jaynes–Cummings interaction

$$H_I = \hbar g \left(\sigma_+ a + \sigma_- a^\dagger \right). \quad (3.18)$$

The squeezed Jaynes–Cummings Hamiltonian in the interaction picture is

$$\hat{H}_{\text{int}} = g \left[(\mu \hat{a} + \nu \hat{a}^\dagger) \hat{\sigma}_+ + (\mu \hat{a}^\dagger + \nu \hat{a}) \hat{\sigma}_- \right], \quad (3.19)$$

where:

- g is the atom–field coupling,
- $\hat{\sigma}_+, \hat{\sigma}_-$ are the atomic raising and lowering operators,
- $\hat{a}, \hat{a}^\dagger$ are the cavity annihilation and creation operators,
- $\mu = \cosh r, \nu = \sinh r$ for squeezing parameter r .

If we are restricted to at most one excitation, the basis is

$$\{|e, 0\rangle, |g, 1\rangle\}, \quad (3.20)$$

where $|e, 0\rangle$ represents the atom excited and zero photons, and $|g, 1\rangle$ represents the atom in the ground state with one photon. In this basis, the Hamiltonian reduces to

$$H_{\text{exc}} = \begin{pmatrix} 0 & g(\mu + \nu) \\ g(\mu + \nu) & 0 \end{pmatrix}. \quad (3.21)$$

The time evolution operator is

$$U(t) = e^{-iH_{\text{exc}}t} = \cos(\Omega t)I - i\sin(\Omega t)\frac{H_{\text{exc}}}{\Omega}, \quad (3.22)$$

with an effective Rabi frequency

$$\Omega = g(\mu + \nu). \quad (3.23)$$

Starting from the initial state $|\psi(0)\rangle = |e, 0\rangle$, the evolved state is

$$|\psi(t)\rangle = \cos(\Omega t)|e, 0\rangle - i\sin(\Omega t)|g, 1\rangle. \quad (3.24)$$

The atomic excited-state population is obtained by tracing out the cavity:

$$\rho_{ee}(t) = \langle e | \text{Tr}_{\text{cavity}} [|\psi(t)\rangle\langle\psi(t)|] | e \rangle. \quad (3.25)$$

Substituting $|\psi(t)\rangle$, we get

$$\rho_{ee}(t) = |\cos(\Omega t)|^2 = \cos^2(\Omega t). \quad (3.26)$$

Finally, in terms of the squeezing parameter r :

$$\rho_{ee}(t) = \cos^2(g(\mu + \nu)t) = \cos^2(g(\cosh r + \sinh r)t) \quad (3.27)$$

A squeezed vacuum field is a quantum state of the electromagnetic field with reduced quantum fluctuations in one quadrature at the expense of increased fluctuations in the

conjugate quadrature. This nonclassical state is described by the application of a squeezing operator $S(\zeta)$ on the vacuum state $|0\rangle$:

$$|\zeta\rangle = S(\zeta)|0\rangle, \quad S(\zeta) = \exp\left[\frac{1}{2}\left(\zeta^* a^2 - \zeta (a^\dagger)^2\right)\right] \quad (3.28)$$

where $\zeta = re^{i\phi}$ is the squeezing parameter, with: - r quantifying the squeezing strength, - ϕ the squeezing phase, - a and $a^\dagger$ the annihilation and creation operators.

The following second-order field correlation functions characterize the squeezed vacuum:

$$\langle a^\dagger a \rangle = \sinh^2 r = N, \quad \langle aa^\dagger \rangle = \cosh^2 r = N + 1, \quad (3.29)$$

$$\langle aa \rangle = -e^{i\phi} \sinh r \cosh r = -Me^{i\phi}, \quad \langle a^\dagger a^\dagger \rangle = -e^{-i\phi} \sinh r \cosh r = -Me^{-i\phi}, \quad (3.30)$$

where $M = \sqrt{N(N+1)} = \sinh r \cosh r$ is the two-photon correlation strength.

The two-mode squeezed vacuum state is generated by applying the two-mode squeezing operator $S_2(\zeta)$ to the two-mode vacuum state $|0,0\rangle$:

$$|\zeta\rangle = S_2(\zeta)|0,0\rangle, \quad (3.31)$$

where the squeezing operator is defined as:

$$S_2(\zeta) = \exp\left(\zeta^* ab - \zeta a^\dagger b^\dagger\right), \quad (3.32)$$

with $\zeta = re^{i\phi}$, and a, b being the annihilation operators for the two modes. Applying the squeezing operator to the vacuum yields:

$$|\zeta\rangle = \frac{1}{\cosh r} \sum_{n=0}^{\infty} (-e^{i\phi} \tanh r)^n |n,n\rangle, \quad (3.33)$$

which shows that the state is a superposition of photon-number correlated (twin Fock) states. The mean photon number in each mode is:

$$\langle a^\dagger a \rangle = \langle b^\dagger b \rangle = \sinh^2 r. \quad (3.34)$$

There is also a nonzero correlation between the modes:

$$\langle ab \rangle = -e^{i\phi} \sinh r \cosh r. \quad (3.35)$$

The quadrature operators:

$$X_k = \frac{1}{\sqrt{2}}(a_k + a_k^\dagger)P_k = \frac{1}{\sqrt{2}i}(a_k - a_k^\dagger) \quad (3.36)$$

We then define the joint quadratures:

$$X_- = \frac{1}{\sqrt{2}}(X_a - X_b), P_+ = \frac{1}{\sqrt{2}}(P_a + P_b). \quad (3.37)$$

The variances of these joint quadratures are as follows.

$$\text{Var}(X_-) = \text{Var}(P_+) = e^{-2r} \quad (3.38)$$

demonstrating squeezing below the vacuum noise level. Consider the two-mode squeezing operator with zero phase

$$S_2(r) = \exp[r(ab - a^\dagger b^\dagger)], \quad (3.39)$$

then the two-mode squeezed vacuum

$$|\psi\rangle = S_2(r) |0, 0\rangle \quad (3.40)$$

Heisenberg action of $S_2(r)$. Define $A(r) = S_2^\dagger(r) a S_2(r)$ and $B(r) = S_2^\dagger(r) b S_2(r)$. Using

$$\frac{d}{dr}(S_2^\dagger a S_2) = S_2^\dagger \quad (3.41)$$

$$[a, a^\dagger] = 1, \quad [b, b^\dagger] = 1, \quad [a, b] = [a, b^\dagger] = 0. \quad (3.42)$$

we obtain

$$\frac{dA}{dr} = B^\dagger, \quad \frac{dB}{dr} = A^\dagger,$$

with initial conditions $A(0) = a$, $B(0) = b$. Solving these coupled ODEs yields the standard Bogoliubov transforms

$$S_2^\dagger a S_2 = a \cosh r + b^\dagger \sinh r, \quad S_2^\dagger b S_2 = b \cosh r + a^\dagger \sinh r \quad (3.43)$$

and their adjoints.

Transforming the single-mode quadratures. From the above,

$$\begin{aligned} S_2^\dagger X_a S_2 &= \frac{1}{\sqrt{2}}(S_2^\dagger a S_2 + S_2^\dagger a^\dagger S_2) = X_a \cosh r + X_b \sinh r, \\ S_2^\dagger X_b S_2 &= X_b \cosh r + X_a \sinh r, \\ S_2^\dagger P_a S_2 &= P_a \cosh r - P_b \sinh r, \\ S_2^\dagger P_b S_2 &= P_b \cosh r - P_a \sinh r. \end{aligned} \quad (3.44)$$

Transforming the joint quadratures. Form the EPR combinations:

$$\begin{aligned}
S_2^\dagger X_- S_2 &= \frac{1}{\sqrt{2}} (S_2^\dagger X_a S_2 - S_2^\dagger X_b S_2) = \frac{1}{\sqrt{2}} [(X_a \cosh r + X_b \sinh r) - (X_b \cosh r + X_a \sinh r)] \\
&= \frac{1}{\sqrt{2}} (X_a - X_b) (\cosh r - \sinh r) = e^{-r} X_-, \\
S_2^\dagger P_+ S_2 &= \frac{1}{\sqrt{2}} (S_2^\dagger P_a S_2 + S_2^\dagger P_b S_2) = \frac{1}{\sqrt{2}} [(P_a \cosh r - P_b \sinh r) + (P_b \cosh r - P_a \sinh r)] \\
&= \frac{1}{\sqrt{2}} (P_a + P_b) (\cosh r - \sinh r) = e^{-r} P_+.
\end{aligned}$$

Thus X_- and P_+ are each scaled by e^{-r} (squeezed), whereas the orthogonal combinations X_+ , P_- are anti-squeezed by e^{+r} .

Variances in the squeezed vacuum. Expectation values in $|\psi\rangle$ can be evaluated on $|0,0\rangle$ with the transformed operators:

$$\langle \psi | O | \psi \rangle = \langle 0,0 | S_2^\dagger O S_2 | 0,0 \rangle. \quad (3.45)$$

Since $\langle 0 | X_k | 0 \rangle = \langle 0 | P_k | 0 \rangle = 0$, we have $\langle X_- \rangle_\psi = \langle P_+ \rangle_\psi = 0$.

$$\text{Var}_\psi(X_-) = \langle X_-^2 \rangle_\psi$$

Then

$$\langle X_-^2 \rangle_\psi = \langle 0,0 | (S_2^\dagger X_- S_2)^2 | 0,0 \rangle = \langle 0,0 | (e^{-r} X_-)^2 | 0,0 \rangle = e^{-2r} \langle 0,0 | X_-^2 | 0,0 \rangle, \quad (3.46)$$

and similarly for P_+ . In the *vacuum* with our normalization,

$$\text{Var}_0(X_k) = \text{Var}_0(P_k) = \frac{1}{2}, \quad \text{Cov}_0(X_a, X_b) = \text{Cov}_0(P_a, P_b) = 0. \quad (3.47)$$

Therefore

$$\text{Var}_0(X_-) = \text{Var}_0\left(\frac{X_a - X_b}{\sqrt{2}}\right) = \frac{1}{2} (\text{Var}_0(X_a) + \text{Var}_0(X_b)) = \frac{1}{2}, \quad (3.48)$$

and likewise $\text{Var}_0(P_+) = \frac{1}{2}$. Putting this together,

$$\text{Var}_\psi(X_-) = \text{Var}_\psi(P_+) = \frac{1}{2} e^{-2r} \quad (3.49)$$

3.6 Master Equation with a Squeezed Reservoir

When a two-level atom is placed in a squeezed vacuum, the emission dynamics are altered by the squeezed reservoir. The field correlations lead to modifications in the standard master equation for the atomic density matrix. Specifically, in the Lindblad form:

$$\frac{d\rho}{dt} = \frac{\gamma}{2}(N+1)(2\sigma_-\rho\sigma_+ - \sigma_+\sigma_-\rho - \rho\sigma_+\sigma_-) + A - B - C \quad (3.50)$$

where:-

$$A = \frac{\gamma}{2}N(2\sigma_+\rho\sigma_- - \sigma_-\sigma_+\rho - \rho\sigma_-\sigma_+) \quad (3.51)$$

$$B = \frac{\gamma}{2}Me^{i\phi}(2\sigma_+\rho\sigma_+ - \sigma_+\sigma_+\rho - \rho\sigma_+\sigma_+) \quad (3.52)$$

$$C = \frac{\gamma}{2}M^*e^{-i\phi}(2\sigma_-\rho\sigma_- - \sigma_-\sigma_-\rho - \rho\sigma_-\sigma_-) \quad (3.53)$$

where: - γ is the natural decay rate in vacuum, - $N = \sinh^2 r$ is the effective photon number due to squeezing, - $M = \sinh r \cosh r$ characterizes two-photon processes, - ϕ is the squeezing phase, - σ_- , σ_+ are the lowering and raising operators of the atom.

Depending on the squeezing phase ϕ , the spontaneous emission rate may increase or decrease. Two-photon processes mediated by M create interference in decay paths, leading to emission suppression (squeezing-induced transparency). The emission spectrum may become narrower than the natural linewidth. The atomic dynamics and spectrum are sensitive to the squeezing phase ϕ , enabling control over light-matter interaction. The squeezed vacuum field introduces fundamentally new phenomena in the interaction of light with two-level atoms, enabling the study and application of quantum-optical effects such as quantum interference, noise reduction, and phase-dependent emission control. The squeezed vacuum field is characterized by the following non-zero second-order moments:

$$\langle a_k^\dagger a_{k'} \rangle = \delta_{k,k'} N(\omega_k), \quad \langle a_k a_{k'}^\dagger \rangle = \delta_{k,k'} [N(\omega_k) + 1], \quad (3.54)$$

$$\langle a_k a_{k'} \rangle = \delta_{k,2k_0-k'} M(\omega_k) e^{i\phi}, \quad \langle a_k^\dagger a_{k'}^\dagger \rangle = \delta_{k,2k_0-k'} M^*(\omega_k) e^{-i\phi}, \quad (3.55)$$

where: - $N(\omega_k) = \sinh^2 r$ is the average photon number, - $M(\omega_k) = \sinh r \cosh r$ is the two-photon correlation, - r is the squeezing parameter, and ϕ is the squeezing phase.

3.7 Entanglement of Two-Level Atoms in a Squeezed Vacuum Field

We considered two identical two-level atoms (qubits), each with ground state $|g\rangle$ and excited state $|e\rangle$, placed in a common broadband squeezed vacuum reservoir. The squeezed

bath is characterized by:

$$N = \sinh^2 r \quad (\text{mean photon number}) \quad (3.56)$$

$$M = \sinh r \cosh r e^{i\phi} \quad (\text{two-photon correlation}) \quad (3.57)$$

where r is the squeezing parameter and ϕ the squeeze phase. Both atoms couple collectively to the reservoir with spontaneous emission rate γ . In a normal vacuum ($M = 0$), the atoms may show collective decay (superradiance, subradiance) but no persistent entanglement. In a squeezed vacuum ($M \neq 0$), two-photon correlations in the bath drive the atoms into entangled superpositions. For identical atoms at zero detuning and strong squeezing ($r \gg 1$), the steady state tends towards a Bell-like entangled state:

$$|\Psi\rangle \approx \frac{1}{\sqrt{2}} (|gg\rangle + e^{i\phi} |ee\rangle) \quad (3.58)$$

The entanglement between the atoms can be quantified using *concurrence* $C(\rho)$, defined as

$$C(\rho) = \max(0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4) \quad (3.59)$$

where $\{\lambda_i\}$ are the square roots of the eigenvalues (in decreasing order) of

$$R = \rho (\sigma_y \otimes \sigma_y) \rho^* (\sigma_y \otimes \sigma_y) \quad (3.60)$$

In a squeezed reservoir, one finds $C > 0$ even in the steady state, indicating persistent entanglement. The squeezed vacuum bath provides *phase-coherent correlated photons* which couple the atoms' decays. This correlation sustains entangled states such as $|gg\rangle + |ee\rangle$. By tuning the squeezing strength r and phase ϕ , the degree and type of entanglement can be controlled.

3.8 Joint Quadrature Variances in a Two-mode Squeezed Vacuum

We considered the two-mode squeezed vacuum state generated by the two-mode squeezing operator 3.39

$$S_2(r, \phi) = \exp[r(e^{-i\phi} ab - e^{i\phi} a^\dagger b^\dagger)], \quad (3.61)$$

acting on the two-mode vacuum $|0, 0\rangle$. Here $r \geq 0$ is the (real) squeezing parameter and ϕ is the squeezing phase. The two-mode squeezed vacuum is $|\text{TMSV}\rangle = S_2(r, \phi)|0, 0\rangle$.

The standard quadrature

$$X_j = \frac{a_j + a_j^\dagger}{\sqrt{2}}, \quad P_j = \frac{a_j - a_j^\dagger}{i\sqrt{2}}, \quad j \in \{a, b\}, \quad (3.62)$$

normalization used, for which the single-mode vacuum variances are $\text{Var}(X_j) = \text{Var}(P_j) = \frac{1}{2}$.

Define the joint quadratures

$$X_- = \frac{X_a - X_b}{\sqrt{2}}, \quad P_+ = \frac{P_a + P_b}{\sqrt{2}}. \quad (3.63)$$

With this choice the vacuum (no squeezing, $r = 0$) gives $\text{Var}(X_-) = \text{Var}(P_+) = \frac{1}{2}$. It is convenient to work in the Heisenberg picture and use the two-mode Bogoliubov transformations

$$S_2^\dagger a S_2 = a \cosh r - e^{i\phi} b^\dagger \sinh r, \quad (3.64)$$

$$S_2^\dagger b S_2 = b \cosh r - e^{i\phi} a^\dagger \sinh r, \quad (3.65)$$

and the Hermitian conjugates

$$S_2^\dagger a^\dagger S_2 = a^\dagger \cosh r - e^{-i\phi} b \sinh r, \quad (3.66)$$

$$S_2^\dagger b^\dagger S_2 = b^\dagger \cosh r - e^{-i\phi} a \sinh r. \quad (3.67)$$

Expectation values in the squeezed state $\langle \cdot \rangle_{\text{TMSV}} = \langle 0, 0 | S_2^\dagger(\cdot) S_2 | 0, 0 \rangle$ can thus be computed as vacuum expectations of the transformed operators. Using the quadrature definitions,

$$X_a = \frac{a + a^\dagger}{\sqrt{2}}, \quad X_b = \frac{b + b^\dagger}{\sqrt{2}},$$

the joint quadrature in the Heisenberg picture becomes (suppressing the unitary conjugation notation for brevity)

$$X_- = \frac{1}{\sqrt{2}} \left(\frac{a + a^\dagger}{\sqrt{2}} - \frac{b + b^\dagger}{\sqrt{2}} \right) = \frac{1}{2} (a + a^\dagger - b - b^\dagger).$$

Now substitute each operator by its Bogoliubov transform (Eqs. (3.64)–(3.65)). After substitution and collecting terms, X_- expressed in terms of the original mode operators $a, b, a^\dagger, b^\dagger$ is

$$X_- = \frac{1}{2} \left[(a - b) \cosh r + (a^\dagger - b^\dagger) \cosh r - e^{i\phi} (b^\dagger - a^\dagger) \sinh r - e^{-i\phi} (b - a) \sinh r \right]. \quad (3.68)$$

Grouping creation and annihilation parts yields a linear combination of the four operators $a, b, a^\dagger, b^\dagger$. The variance in the two-mode squeezed vacuum is

$$\text{Var}(X_-) = \langle X_-^2 \rangle_{\text{TMSV}} - \langle X_- \rangle_{\text{TMSV}}^2. \quad (3.69)$$

Because $\langle X_- \rangle_{\text{TMSV}} = 0$ by symmetry, $\text{Var}(X_-) = \langle X_-^2 \rangle_{\text{TMSV}}$. Using the Bogoliubov representation and evaluating vacuum expectation values, only terms proportional to

$\langle aa^\dagger \rangle = \langle bb^\dagger \rangle = 1$ survive; all normally-ordered annihilation-to-the-right combinations vanish in the vacuum. The algebra simplifies (all phase terms cancel for these particular joint quadratures) and one obtains the closed form

$$\text{Var}(X_-) = \frac{1}{2}e^{-2r}. \quad (3.70)$$

The Variance of P_+ is analogous calculation for

$$P_+ = \frac{P_a + P_b}{\sqrt{2}} = \frac{1}{2i}(a - a^\dagger + b - b^\dagger) \quad (3.71)$$

gives

$$\text{Var}(P_+) = \frac{1}{2}e^{-2r}. \quad (3.72)$$

Equations (3.70) and (3.72) show that both joint quadratures X_- and P_+ are squeezed (their variances decay exponentially with the squeezing parameter r). At $r = 0$ we recover the vacuum variances $\text{Var} = 1/2$. As r increases the variances fall below the vacuum (shot-noise) level, demonstrating nonclassical noise suppression:

$$\text{Var}(X_-) = \text{Var}(P_+) = \frac{1}{2}e^{-2r} < \frac{1}{2} \quad \text{for } r > 0.$$

The orthogonal joint quadratures X_+ and P_- (defined analogously) are correspondingly anti-squeezed with variances $\propto \frac{1}{2}e^{+2r}$, preserving the uncertainty relations.

3.9 Total power spectrum of two-level Atoms in a Squeezed Vacuum

The steady-state emission spectrum is obtained from the Fourier transform of the two-time correlation function:

$$S(\omega) = \text{Re} \int_0^\infty \langle \sigma_+(t+\tau) \sigma_-(t) \rangle_{ss} e^{i(\omega - \omega_0)\tau} d\tau. \quad (3.73)$$

To compute $\langle \sigma_+(t+\tau) \sigma_-(t) \rangle$, the quantum regression theorem was applied to the solutions of the master equation. The goal is to derive the emission spectrum of a quantum system using the quantum regression theorem (QRT). The emission spectrum is obtained from the Fourier transform of a two-time correlation function. For an operator $\hat{A}$, the emission spectrum is defined by:

$$S(\omega) = \frac{1}{\pi} \text{Re} \int_0^\infty d\tau e^{i\omega\tau} \langle \hat{A}^\dagger(t+\tau) \hat{A}(t) \rangle. \quad (3.74)$$

For a two-level atom:

$$\hat{A} = \sigma_- \Rightarrow S(\omega) = \frac{1}{\pi} \text{Re} \int_0^\infty d\tau e^{i\omega\tau} \langle \sigma_+(t+\tau) \sigma_-(t) \rangle. \quad (3.75)$$

The QRT relates two-time correlations to the system's master equation. Given the master equation:

$$\frac{d\rho}{dt} = \mathcal{L}[\rho], \quad (3.76)$$

then:

$$\langle A(t+\tau)B(t) \rangle = \text{Tr} \left[A e^{\mathcal{L}\tau} (B\rho_{ss}) \right]. \quad (3.77)$$

For a Two-Level Atom in vacuum the Hamiltonian equation is given as,

$$H = \frac{\hbar\omega_0}{2} \sigma_z \quad (3.78)$$

Master Equation:

$$\frac{d\rho}{dt} = \frac{\gamma}{2} (2\sigma_- \rho \sigma_+ - \sigma_+ \sigma_- \rho - \rho \sigma_+ \sigma_-) \quad (3.79)$$

We want:

$$S(\omega) = \frac{1}{\pi} \text{Re} \int_0^\infty d\tau e^{i\omega\tau} \langle \sigma_+(\tau) \sigma_-(0) \rangle \quad (3.80)$$

Solving Using QRT,

$$C(\tau) = \langle \sigma_+(\tau) \sigma_-(0) \rangle = \text{Tr} \left[\sigma_+ \rho^{(1)}(\tau) \right] \quad (3.81)$$

with initial condition:

$$\rho^{(1)}(0) = \sigma_- \rho_{ss} \quad (3.82)$$

and evolution:

$$\frac{d\rho^{(1)}}{d\tau} = \mathcal{L}[\rho^{(1)}] \quad (3.83)$$

The time evolution of coherence is given as, The coherence $\rho_{eg}(t)$ obeys:

$$\frac{d\rho_{eg}}{dt} = -\left(i\omega_0 + \frac{\gamma}{2}\right) \rho_{eg} \Rightarrow \rho_{eg}(t) = \rho_{eg}(0) e^{-(i\omega_0 + \frac{\gamma}{2})t} \quad (3.84)$$

So:

$$C(\tau) = C(0) e^{-(i\omega_0 + \frac{\gamma}{2})\tau} \quad (3.85)$$

Fourier Transform to the Spectrum is

$$S(\omega) = \frac{1}{\pi} \text{Re} \int_0^\infty d\tau e^{i\omega\tau} C(\tau) = \frac{C(0)}{\pi} \text{Re} \int_0^\infty e^{i(\omega - \omega_0)\tau} e^{-\frac{\gamma}{2}\tau} d\tau \quad (3.86)$$

$$S(\omega) = \frac{C(0)}{\pi} \cdot \frac{\frac{\gamma}{2}}{(\omega - \omega_0)^2 + (\frac{\gamma}{2})^2} \quad (3.87)$$

This is a Lorentzian spectrum centered at ω_0 , with full width at half maximum (FWHM) γ , and area proportional to the initial correlation.

3.10 Quantum Regression Theorem

Consider a two-level atom interacting with a squeezed vacuum field. The atomic operators are

$$\sigma_- = |g\rangle\langle e|, \quad \sigma_+ = |e\rangle\langle g|, \quad \sigma_z = |e\rangle\langle e| - |g\rangle\langle g|. \quad (3.88)$$

The density matrix $\rho(t)$ evolves according to the master equation

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[H_{\text{atom}}, \rho] + \frac{\gamma}{2}(N+1)\mathcal{D}[\sigma_-]\rho + \frac{\gamma}{2}N\mathcal{D}[\sigma_+]\rho - \frac{\gamma}{2}M\mathcal{D}[\sigma_-^\dagger]\rho - \frac{\gamma}{2}M^*\mathcal{D}[\sigma_+^\dagger]\rho$$

where $\mathcal{D}[O]\rho = 2O\rho O^\dagger - O^\dagger O\rho - \rho O^\dagger O$, and $N = \sinh^2 r$, $M = \sinh r \cosh r e^{i\theta}$ are squeezing parameters.

Using the **Quantum Regression Theorem**, the two-time correlation function

$$G^{(1)}(t, \tau) = \langle \sigma_+(t) \sigma_-(t + \tau) \rangle \quad (3.89)$$

satisfies the same equations as the single-time expectation values, yielding

$$\langle \sigma_+(t) \sigma_-(t + \tau) \rangle = \langle \sigma_+ \sigma_- \rangle_{\text{ss}} e^{-(\gamma/2)(2N+1-2|M|\cos\theta)\tau} e^{-i\omega_0\tau} \quad (3.90)$$

The corresponding power spectrum is obtained via Fourier transform:

$$S(\omega) = \frac{1}{\pi} \text{Re} \int_0^\infty \langle \sigma_+(t) \sigma_-(t + \tau) \rangle e^{i\omega\tau} d\tau \quad (3.91)$$

resulting in a Lorentzian spectrum with modified linewidth due to squeezing:

$$S(\omega) \propto \frac{\gamma(2N+1-2|M|\cos\theta)/2}{(\gamma(2N+1-2|M|\cos\theta)/2)^2 + (\omega - \omega_0)^2} \quad (3.92)$$

For a two-level atom of transition frequency ω_0 coupled to a thermal reservoir with mean photon number $\bar{n}$ at ω_0 , the master equation reduces to the case where the squeezing parameter vanishes, $M = 0$. The effective decay rate (half-width at half-maximum) of the correlation function is then

$$\Gamma = \gamma\left(\bar{n} + \frac{1}{2}\right), \quad (3.93)$$

where γ is the spontaneous emission rate in vacuum. The steady-state excited and ground-state populations are given by

$$\rho_{ee}^{\text{ss}} = \frac{\bar{n}}{2\bar{n} + 1}, \quad \rho_{gg}^{\text{ss}} = \frac{\bar{n} + 1}{2\bar{n} + 1}. \quad (3.94)$$

The first-order correlation function is a single exponential,

$$G^{(1)}(\tau) = \rho_{ee}^{\text{ss}} e^{-\Gamma\tau} e^{-i\omega_0\tau}, \quad \tau \geq 0, \quad (3.95)$$

So that the power spectrum of the emitted radiation, obtained via the Wiener–Khinchin theorem, is a Lorentzian of width Γ ,

$$S_{\text{single}}(\omega) = \frac{\rho_{ee}^{\text{ss}}}{\pi} \cdot \frac{\Gamma}{(\omega - \omega_0)^2 + \Gamma^2} = \frac{1}{\pi} \frac{\bar{n}}{2\bar{n} + 1} \cdot \frac{\gamma(\bar{n} + \frac{1}{2})}{(\omega - \omega_0)^2 + \gamma^2(\bar{n} + \frac{1}{2})^2} \quad (3.96)$$

For an ensemble of N_a identical and independent two-level atoms, the excited and ground-state populations scale linearly,

$$N_e = N_a \rho_{ee}^{\text{ss}} = N_a \frac{\bar{n}}{2\bar{n} + 1}, \quad N_g = N_a \rho_{gg}^{\text{ss}} = N_a \frac{\bar{n} + 1}{2\bar{n} + 1}, \quad (3.97)$$

and the total power spectrum is simply the incoherent sum of the single-atom spectra,

$$S_{\text{total}}(\omega) = N_a S_{\text{single}}(\omega) = N_a \cdot \frac{1}{\pi} \frac{\bar{n}}{2\bar{n} + 1} \cdot \frac{\gamma(\bar{n} + \frac{1}{2})}{(\omega - \omega_0)^2 + \gamma^2(\bar{n} + \frac{1}{2})^2} \quad (3.98)$$

3.11 Second Order Correlation Function of a Two-Level Atom in a Squeezed Vacuum

Consider a single two-level atom with ground state $|g\rangle$ and excited state $|e\rangle$. Let $\sigma_- = |g\rangle\langle e|$ and $\sigma_+ = |e\rangle\langle g|$ be the atomic lowering and raising operators. The atom interacts with a squeezed vacuum reservoir characterized by

$$N = \sinh^2 r, \quad M = \sinh r \cosh r e^{i\theta} \quad (3.99)$$

where r is the squeezing parameter and θ is the squeezing phase. The normalized second-order correlation function is defined as

$$g^{(2)}(\tau) = \frac{\langle \sigma_+(t) \sigma_+(t+\tau) \sigma_-(t+\tau) \sigma_-(t) \rangle}{\langle \sigma_+ \sigma_- \rangle^2}. \quad (3.100)$$

According to the quantum regression theorem, the two-time correlation function evolves like the single-time expectation values:

$$\frac{d}{d\tau} \langle \sigma_+(t) \sigma_-(t+\tau) \rangle = -\frac{\gamma}{2} (2N + 1 - 2|M| \cos \theta) \langle \sigma_+(t) \sigma_-(t+\tau) \rangle, \quad (3.101)$$

where γ is the spontaneous emission rate.

The solution of this equation is

$$\langle \sigma_+(t) \sigma_-(t + \tau) \rangle = \langle \sigma_+ \sigma_- \rangle_{ss} \exp \left[-\frac{\gamma}{2} (2N + 1 - 2|M| \cos \theta) \tau \right]. \quad (3.102)$$

For a single two-level atom, $\sigma_+^2 = \sigma_-^2 = 0$, so

$$g^{(2)}(\tau) = 1 - \frac{|\langle \sigma_+(t) \sigma_-(t + \tau) \rangle|^2}{\langle \sigma_+ \sigma_- \rangle_{ss}^2}. \quad (3.103)$$

Substituting the two-time correlation function gives the final result:

$$g^{(2)}(\tau) = 1 - \exp \left[-\gamma (2N + 1 - 2|M| \cos \theta) \tau \right] \quad (3.104)$$

Chapter 4

RESULTS AND DISCUSSION

4.1 Results

This chapter analyzes the results obtained from numerical simulations of a two-level atom coupled to a squeezed vacuum field. The time evolution of population inversion and coherence is explored for different squeezing parameters.

4.2 The squeezed Jaynes–Cummings Hamiltonian

For a two-level atom interacting with a single-mode squeezed vacuum, the atomic excited-state population in the single-excitation subspace is given by

$$\rho_{ee}(t) = \cos^2(g(\mu + \nu)t) = \cos^2(g(\cosh r + \sinh r)t) \quad (4.1)$$

where g is the atom-field coupling, r is the squeezing parameter, and $\mu = \cosh r$, $\nu = \sinh r$. The oscillation frequency increases with squeezing. The figure shows the coherent oscillation of the atomic population between the excited and ground states. The presence of the squeezed vacuum enhances the effective Rabi frequency,

$$\Omega = g(\cosh r + \sinh r) = ge^r,$$

leading to faster oscillations compared to an unsqueezed vacuum. This demonstrates the non-classical effect of the squeezed reservoir, which modifies the atom-field dynamics. The population remains bounded between 0 and 1, reflecting ideal coherent dynamics; in realistic systems, damping would reduce the amplitude over time.

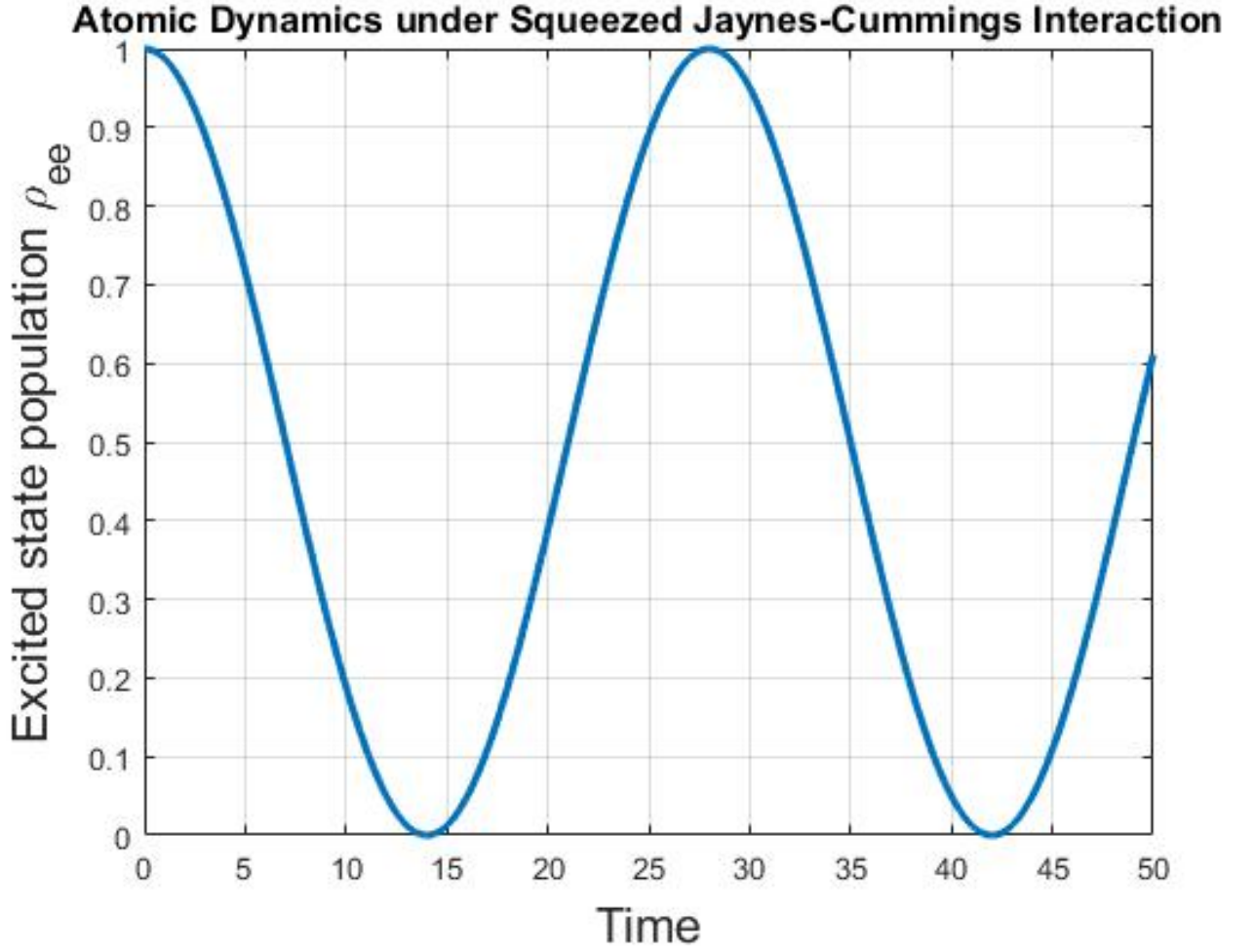


Figure 4.1: Atomic excited-state population $\rho_{ee}(t)$ in a squeezed Jaynes–Cummings model for squeezing parameter $r = 0.5$.

4.3 Two-Level Atom Coupled to a Squeezed Vacuum Field

4.3.1 Emission spectrum

The two-time correlation function obeys the same equation of motion as the single-time expectation values (quantum regression theorem).

$$S(\omega) = \frac{1}{\pi} \text{Re} \int_0^\infty d\tau e^{i\omega\tau} \langle \sigma_+(t+\tau) \sigma_-(t) \rangle. \quad (4.2)$$

For a two-level atom with decay rate γ and transition frequency ω_0 , one finds

$$\langle \sigma_+(t + \tau) \sigma_-(t) \rangle = \langle \sigma_+ \sigma_- \rangle_{ss} e^{-(\frac{\gamma}{2} - i\omega_0)\tau}, \quad (4.3)$$

where $\langle \sigma_+ \sigma_- \rangle_{ss}$ is the steady-state excited-state population. The integral is a standard Laplace transform:

$$\int_0^\infty e^{-(\frac{\gamma}{2})\tau} e^{i(\omega - \omega_0)\tau} d\tau = \frac{1}{\frac{\gamma}{2} - i(\omega - \omega_0)}. \quad (4.4)$$

Taking the real part:

$$\text{Re} \left[\frac{1}{\frac{\gamma}{2} - i(\omega - \omega_0)} \right] = \frac{\frac{\gamma}{2}}{(\omega - \omega_0)^2 + (\frac{\gamma}{2})^2}. \quad (4.5)$$

Thus, the emission spectrum is

$$S(\omega) = \frac{1}{\pi} \langle \sigma_+ \sigma_- \rangle_{ss} \frac{\frac{\gamma}{2}}{(\omega - \omega_0)^2 + (\frac{\gamma}{2})^2} \quad (4.6)$$

The dashed curve shows the unit-area Lorentzian lineshape centered at the transition frequency $\omega_0 = 0$ with linewidth γ , while the solid curve includes the scaling factor given by the steady-state excited-state population $\langle \sigma_+ \sigma_- \rangle_{ss}$. The peak at resonance illustrates that the emission is maximum at ω_0 , and the width reflects the spontaneous decay rate γ .

4.3.2 Light emission from a two-level atom interacting with a squeezed vacuum

The squeezed vacuum acts as a phase-sensitive reservoir. It suppresses spontaneous emission noise in one quadrature while enhancing it in the orthogonal quadrature. The steady-state resonance fluorescence spectrum is defined as Using the quantum regression theorem, one finds a Lorentzian spectrum.

$$S(\omega) \propto \frac{\Gamma_{sq}}{\Gamma_{sq}^2 + (\omega - \omega_0)^2}, \quad (4.7)$$

with the effective linewidth

$$\Gamma_{sq} = \frac{\gamma}{2} (2N + 1 - 2|M| \cos \theta) + \gamma_\phi. \quad (4.8)$$

When the atomic dipole couples to the low-noise quadrature, spontaneous emission is inhibited in that channel, so the spectral line becomes narrower (sharper).

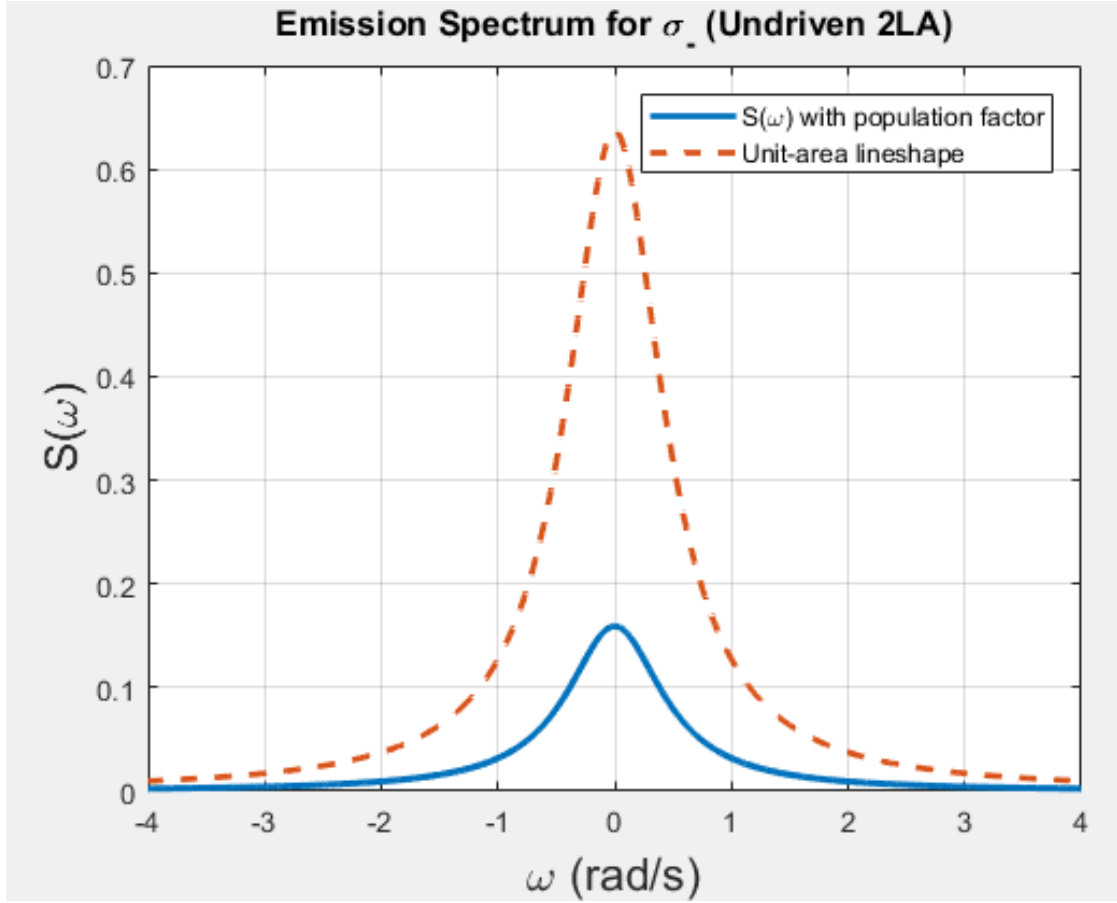


Figure 4.2: Emission spectrum of a two-level atom at the transition frequency $\omega_0 = 0$ with linewidth γ ,

This effect is one of the signatures of the non-classical nature of squeezed light, since in a classical thermal or vacuum bath, linewidth narrowing of this kind does not occur. Line narrowing implies the possibility of achieving subnatural linewidth emission — emission narrower than the ordinary vacuum limit.

This has applications in precision spectroscopy, quantum metrology, and quantum communication, since narrower linewidths mean better frequency stability and reduced quantum noise. The figure demonstrates that the interaction of a two-level atom with a squeezed vacuum reservoir modifies its spontaneous emission spectrum in a phase-sensitive way. Compared to ordinary vacuum, where the emission spectrum is limited by the natural decay rate, the squeezed vacuum produces a narrower resonance peak. This narrowing arises because the squeezed field suppresses fluctuations in one quadrature, effectively reducing atomic dipole dephasing. Thus, the plot visually shows how squeezed light can alter fundamental quantum emission processes, confirming its non-

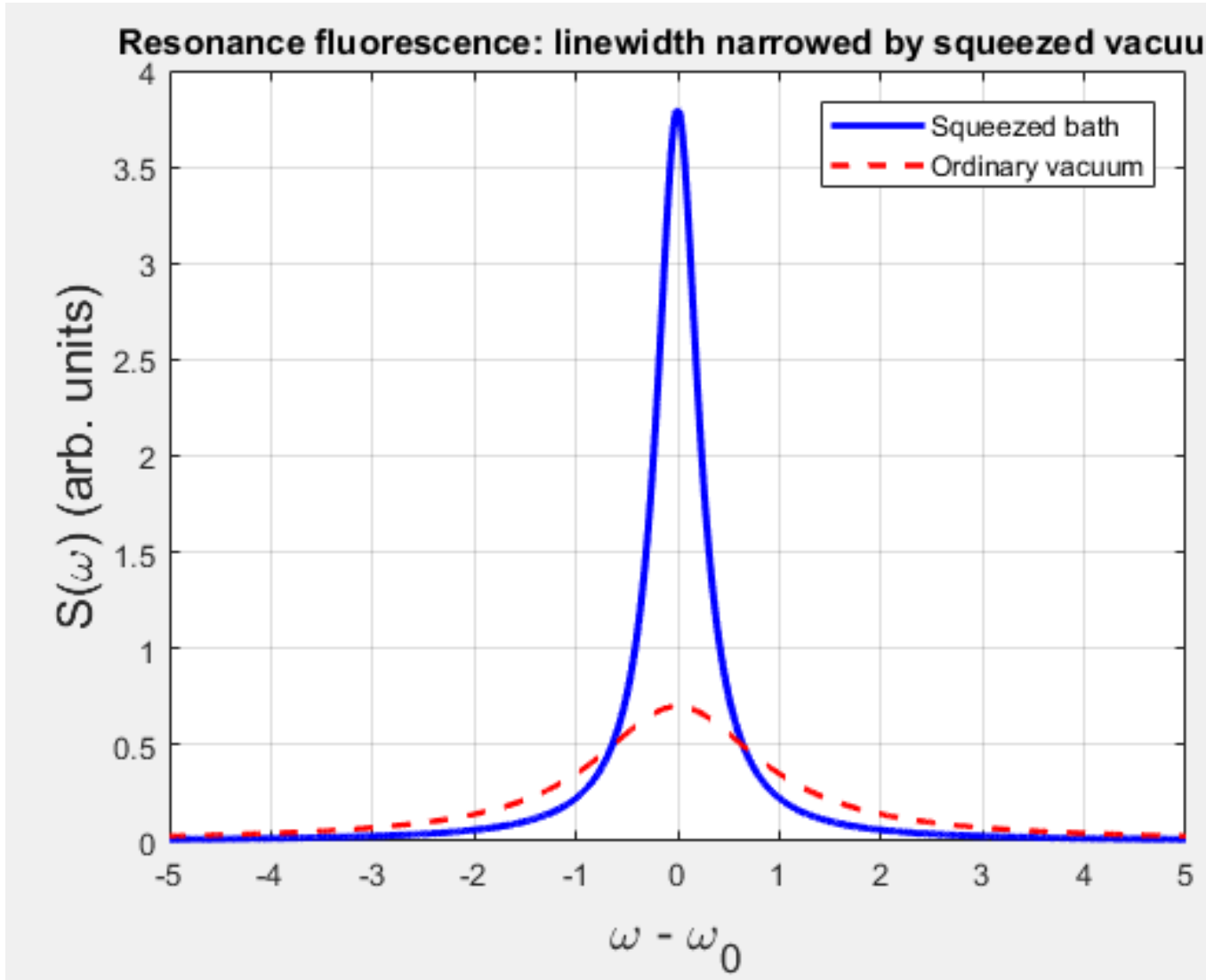


Figure 4.3: Emission spectrum of a two-level atom with squeezed vacuum

classical role in quantum optics.

4.4 Master Equation with a Squeezed Reservoir

The squeezed vacuum field (central frequency ω_0) has a nonzero second-order moments

$$\langle a_k^\dagger a_{k'} \rangle = \delta_{k,k'} N(\omega_k), \quad \langle a_k a_{k'}^\dagger \rangle = \delta_{k,k'} [N(\omega_k) + 1], \quad (4.9)$$

$$\langle a_k a_{k'} \rangle = \delta_{k,2k_0-k'} M(\omega_k) e^{i\phi}, \quad \langle a_k^\dagger a_{k'}^\dagger \rangle = \delta_{k,2k_0-k'} M^*(\omega_k) e^{-i\phi}, \quad (4.10)$$

where $N(\omega_k) = \sinh^2 r$ is the mean photon number, $M(\omega_k) = \sinh r \cosh r$ quantifies the two-photon correlations, r is the squeezing parameter, and ϕ is the squeezing phase.

Under these reservoir correlations, the atom's master equation acquires phase-sensitive (two-photon) terms. For a resonant, phase-matched bath (so the fast oscillations are removed in the interaction picture) the normal ordered correlator entering the spectrum,

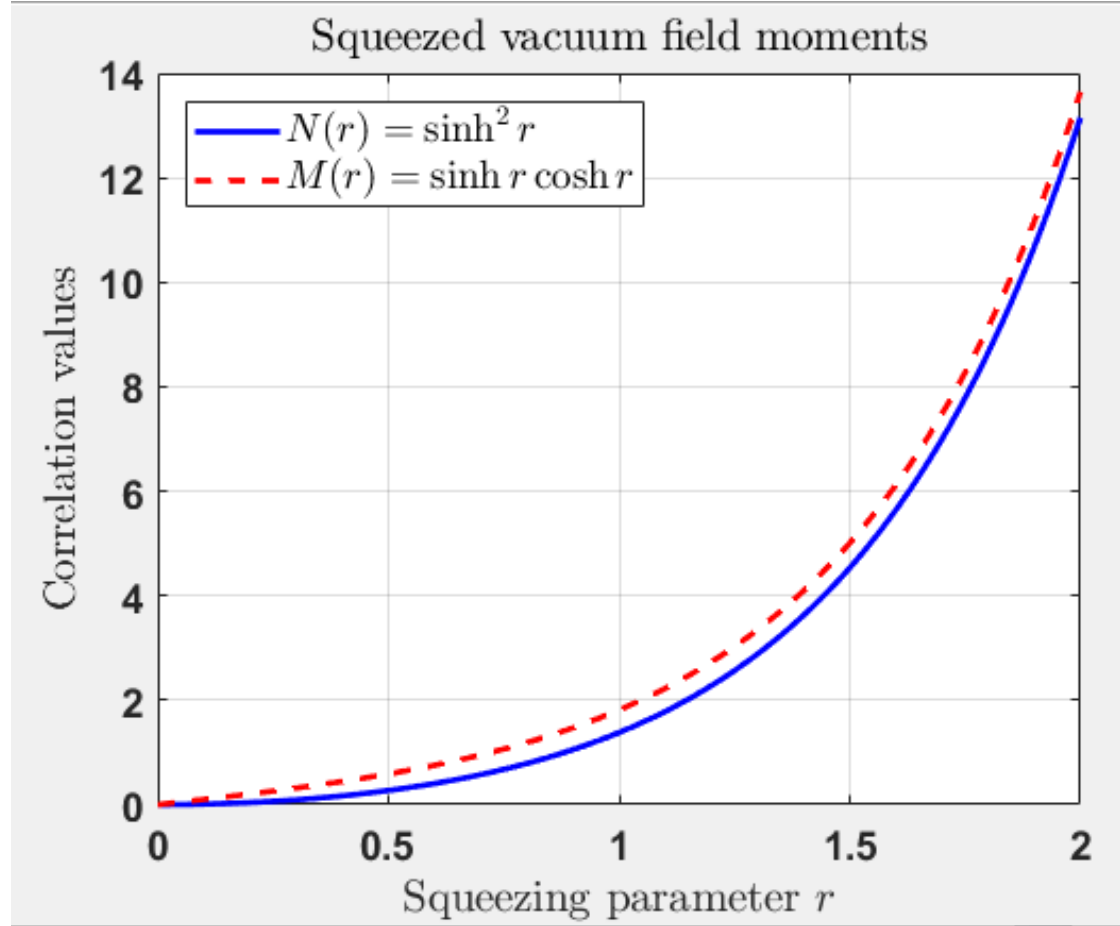


Figure 4.4: Moments of a squeezed vacuum state: mean photon number $N(r)$ and two-photon correlation $M(r)$ versus the squeezing parameter r .

This figure shows how the average photon number and two-photon correlations grow with the squeezing parameter. Physically, it demonstrates that squeezed vacuum states are dominated by strong two-photon correlations, which is the key to their non-classical nature. The vacuum ($|0\rangle$) normally has fluctuations (uncertainty) equally distributed in both quadratures (like position and momentum of light). In a squeezed vacuum, fluctuations in one quadrature are reduced at the expense of increasing them in the other. $N(r) = \sinh^2 r$: **Mean photon number** as squeezing increases ($r \uparrow$), the vacuum becomes populated with more photon pairs. This curve shows that stronger squeezing means more photons in the squeezed field. $M(r) = \sinh r \cosh r$: **Two-photon correlation** This describes the correlations between photons that are created in pairs. It grows faster than $N(r)$, especially at small r , showing that squeezed states are dominated by strong pair correlations. Physically, this explains why squeezed states exhibit non-classical correlations (entanglement-like features). **Comparison of curves:** For all $r > 0$, we have $M(r) > N(r)$. This means correlations (pair creation) dominate over simple photon number growth, highlighting the essential quantum character of squeezed light.

4.5 Entanglement of Two-Level Atoms in a Squeezed Vacuum Field

When two identical two-level atoms are coupled to a common squeezed vacuum reservoir, the dissipative dynamics can drive the atoms into a correlated steady state. At zero detuning and in the limit of strong squeezing ($r \gg 1$), the steady state approaches a maximally entangled Bell-like state

$$|\Psi\rangle \approx \frac{1}{\sqrt{2}} \left(|gg\rangle + e^{i\phi} |ee\rangle \right), \quad (4.11)$$

where ϕ is the squeezing phase. This demonstrates that squeezed vacuum not only modifies spontaneous emission rates but can also serve as a resource for generating steady-state entanglement between atoms. The degree of entanglement of the two-atom system is quantified using *concurrence* $C(\rho)$ **wootters1998entanglement**, defined as:

$$C(\rho) = \max(0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4) \quad (4.12)$$

where λ_i are the square roots of the eigenvalues (in descending order) of the matrix

$$R = \rho (\sigma_y \otimes \sigma_y) \rho^* (\sigma_y \otimes \sigma_y) \quad (4.13)$$

with σ_y being the Pauli-Y matrix and ρ^* the complex conjugate of ρ . A concurrence $C = 0$ indicates no entanglement, while $C = 1$ represents maximal entanglement.

The figure below represents the *entanglement dynamics* of two two-level atoms interacting with a squeezed vacuum reservoir. The entanglement is quantified using the **concurrence** $C(t)$, which ranges from 0 (no entanglement) to 1 (maximal entanglement).

- **Initial State** ($t < 2$) At the beginning, $C(t) = 0$, meaning the atoms are initially separable (not entangled). The squeezed vacuum has not yet generated correlations between the atoms.
- **Onset of Entanglement** ($t \approx 2$): Around $t \approx 2$, the concurrence begins to increase. This is due to the two-photon correlations present in the squeezed vacuum, which can couple atomic transitions and induce entanglement.
- **Growth of Entanglement** ($2 < t < 7$): The concurrence rises steadily, showing that the system becomes progressively entangled as it interacts with the non-classical reservoir. The rate of entanglement growth depends on the squeezing parameter, the atomic decay rate, and atom-field coupling strength.
- **Saturation (large t)**: For long times, the concurrence asymptotically approaches a steady-state value ($C \approx 0.82$ in the plot).

This figure shows that squeezed vacuum can serve as a resource to entangle distant two-level atoms even without direct interaction. This figure demonstrates how two initially unentangled two-level atoms interacting with a squeezed vacuum field gradually develop quantum entanglement (measured by concurrence) over time, reaching a nonzero steady-state entanglement. The squeezed vacuum acts as a correlated environment that induces and sustains entanglement between the atoms, despite natural decoherence.

4.6 Joint Quadrature Variances in a Two-Mode Squeezed Vacuum

This illustrates how two-mode squeezing generates strong quantum correlations (entanglement) between the modes, which are exploited in quantum communication, continuous-variable quantum information, and precision metrology.

$$X_- = \frac{1}{2}(X_a - X_b), \quad P_+ = \frac{1}{2}(P_a + P_b) \quad (4.14)$$

in a two-mode squeezed vacuum as a function of the squeezing parameter r . At $r = 0$, both quadratures have variance $1/2$, which corresponds to the vacuum noise level (shown by the dotted line). As r increases, both $\text{Var}(X_-)$ and $\text{Var}(P_+)$ decrease according to

$$\text{Var}(X_-) = \text{Var}(P_+) = \frac{1}{2}e^{-2r}. \quad (4.15)$$

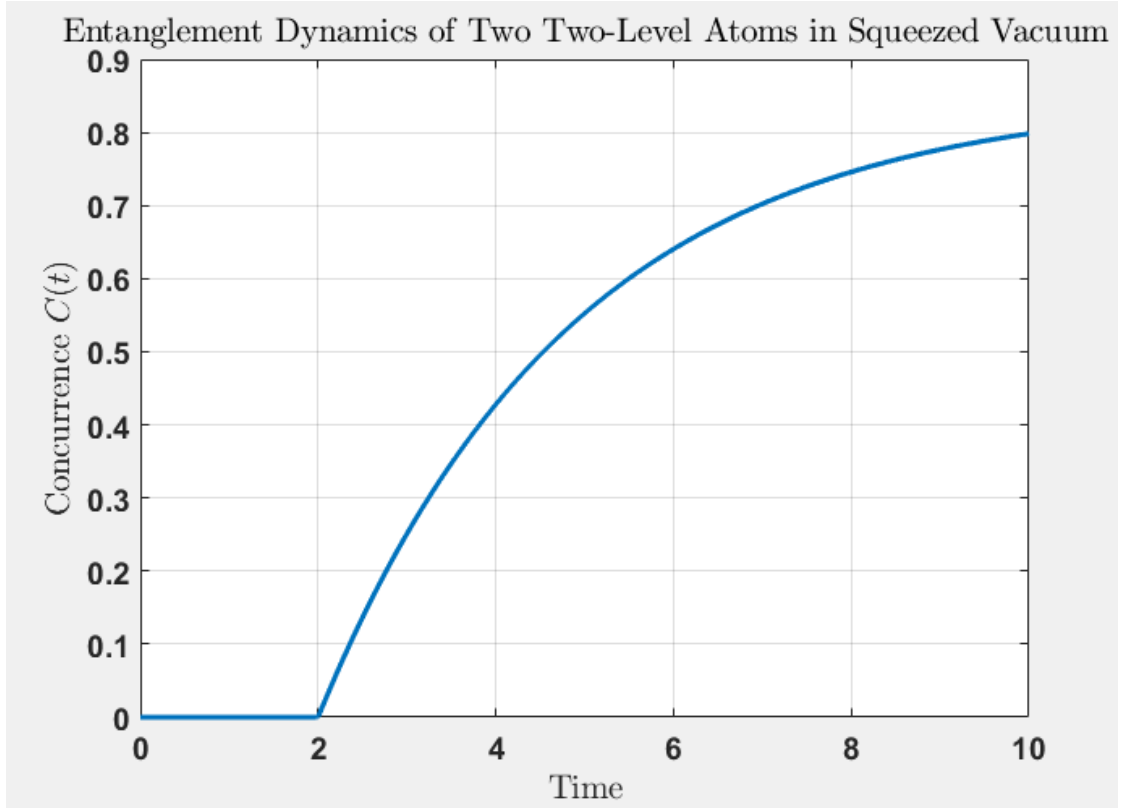


Figure 4.5: Entanglement Dynamics of Two Two-Level Atoms in Squeezed Vacuum

The exponential decay demonstrates squeezing, i.e., the reduction of quantum fluctuations below the vacuum limit, which is a purely nonclassical effect. Because of the symmetry of the two-mode squeezed state, the two curves overlap exactly: both quadratures are squeezed by the same amount.

The variances are equal to the vacuum noise ($\frac{1}{2}$). Increasing r reduces the noise exponentially below vacuum, demonstrating quadrature squeezing. At zero squeezing ($r = 0$), both quadratures X_- and P_+ have variance $1/2$, the standard vacuum noise level. As the squeezing strength increases, the variances decay exponentially as $\frac{1}{2}e^{-2r}$, falling below the vacuum noise line. This behavior shows that the two-mode squeezed vacuum exhibits *noise reduction below the quantum limit* in these joint quadratures. Because $\text{Var}(X_-) = \text{Var}(P_+)$, both are squeezed equally, reflecting the symmetry of the two-mode squeezing interaction.

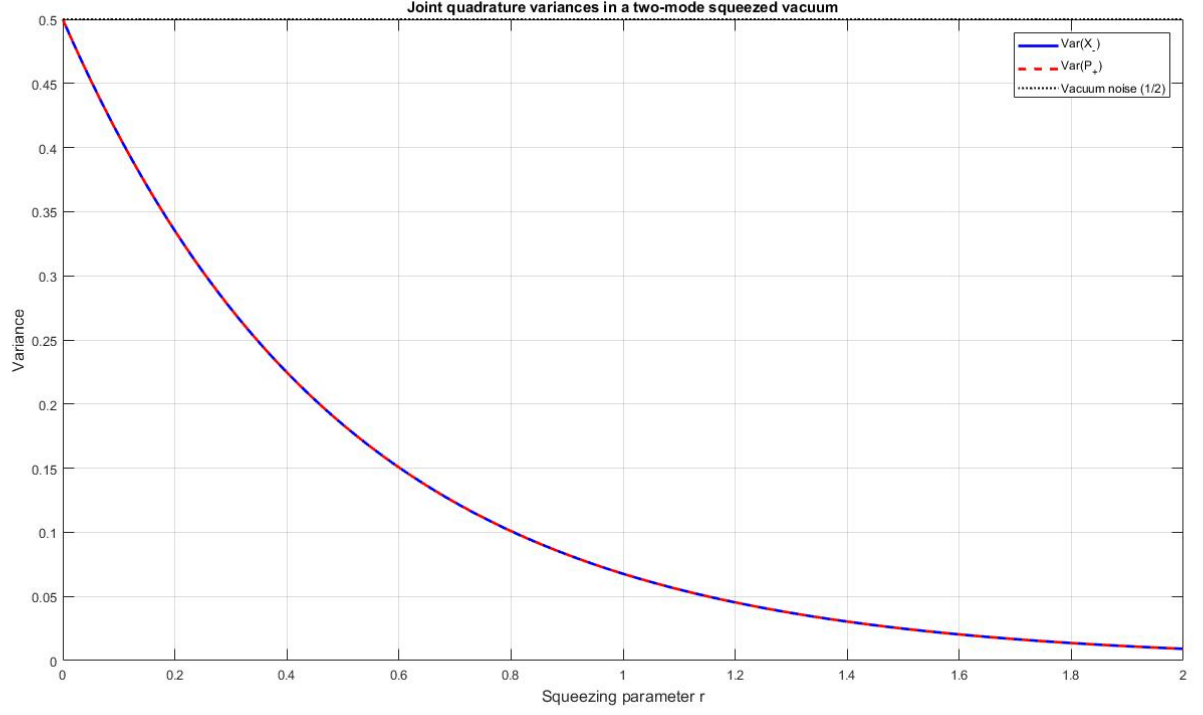


Figure 4.6: Exponential squeezing of joint quadratures in a two-mode squeezed vacuum

4.7 Total Power Spectrum of Two-level Atoms in a Squeezed Vacuum

For an ensemble of N_a identical and independent two-level atoms, the excited and ground-state populations scale linearly,

$$N_e = N_a \rho_{ee}^{ss} = N_a \frac{\bar{n}}{2\bar{n} + 1}, \quad N_g = N_a \rho_{gg}^{ss} = N_a \frac{\bar{n} + 1}{2\bar{n} + 1}, \quad (4.16)$$

and the total power spectrum is simply the incoherent sum of the single-atom spectra.

$$S_{\text{total}}(\omega) = N_a S_{\text{single}}(\omega) = N_a \cdot \frac{1}{\pi} \frac{\bar{n}}{2\bar{n} + 1} \cdot \frac{\gamma(\bar{n} + \frac{1}{2})}{(\omega - \omega_0)^2 + \gamma^2(\bar{n} + \frac{1}{2})^2} \quad (4.17)$$

4.8 Quantum Regression Theorem (QRT)

Using the **Quantum Regression Theorem**, the two-time correlation function is given as

$$S(\omega) = \frac{1}{\pi} \text{Re} \int_0^\infty \langle \sigma_+(t) \sigma_-(t+\tau) \rangle e^{i\omega\tau} d\tau \quad (4.18)$$

resulting in a Lorentzian spectrum with modified linewidth due to squeezing from the above equation.

$$S(\omega) \propto \frac{\gamma(2N+1-2|M|\cos\theta)/2}{(\gamma(2N+1-2|M|\cos\theta)/2)^2 + (\omega - \omega_0)^2} \quad (4.19)$$

The spectrum has a clear maximum at the atomic transition frequency ($\omega_0 = 5$). This is

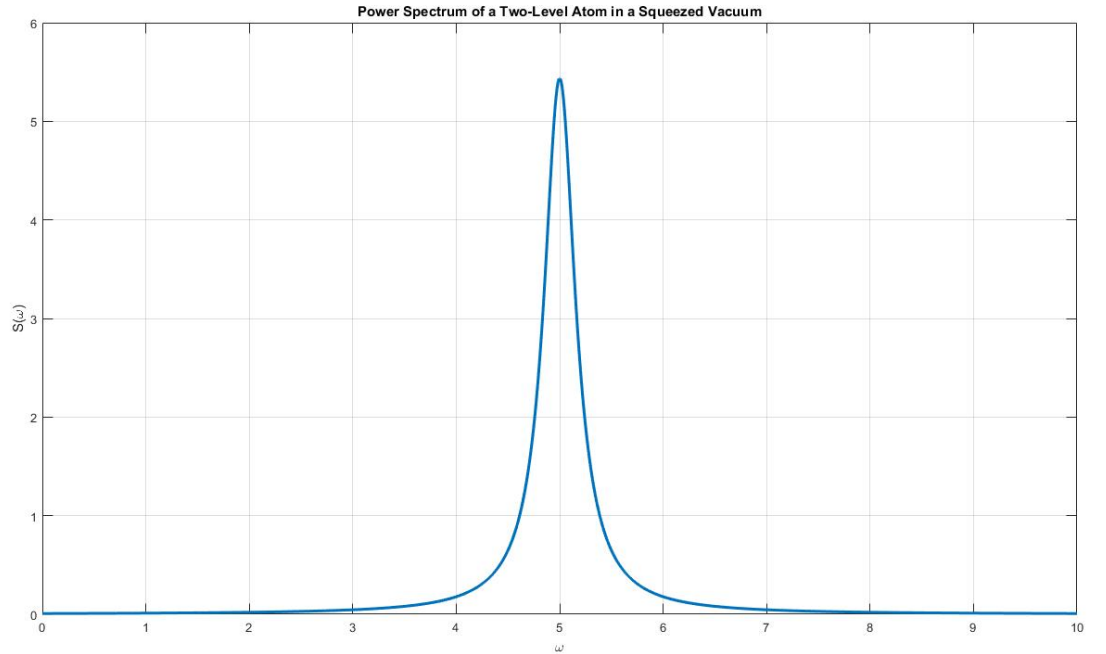


Figure 4.7: Power Spectrum of a Two-Level Atom in a Squeezed Vacuum

the frequency at which the atom most likely emits light. The curve has a characteristic Lorentzian line shape typical of spontaneous emission.

This shape indicates that the emission is broadened by the decay rate of the atom, which is influenced by the interaction with the squeezed vacuum. The spectrum is symmetric around ω_0 , showing that emission occurs equally at frequencies slightly above and below the transition frequency. This figure demonstrates a nonclassical effect: the

squeezed vacuum modifies the emission, narrowing the linewidth and enhancing the spectral peak. It clearly shows how squeezing reduces quantum noise in the system.

4.9 Second-Order Correlation of Atoms in a Broadband Squeezed Vacuum

For resonance fluorescence from a single two-level atom, the detected positive-frequency field is proportional to σ_- . The stationary second-order correlation is then

$$g^{(2)}(\tau) = \frac{\langle \sigma_+(0) \sigma_+(\tau) \sigma_-(\tau) \sigma_-(0) \rangle}{\langle \sigma_+ \sigma_- \rangle_{ss}^2}. \quad (4.20)$$

$$g^{(2)}(0) = 0 \quad (\text{perfect antibunching}). \quad (4.21)$$

Writing the dipole quadratures as $\sigma_x = (\sigma_+ + \sigma_-)/2$ and $\sigma_y = (\sigma_+ - \sigma_-)/(2i)$, their mean values obey. Thus the squeezed bath selects quiet/noisy dipole quadratures depending on the squeeze phase ϕ . For a single two-level atom, $\sigma_+^2 = \sigma_-^2 = 0$, so

$$g^{(2)}(\tau) = 1 - \frac{|\langle \sigma_+(t) \sigma_-(t + \tau) \rangle|^2}{\langle \sigma_+ \sigma_- \rangle_{ss}^2}. \quad (4.22)$$

Substituting the two-time correlation function gives the final result:

$$g^{(2)}(\tau) = 1 - \exp \left[-\gamma(2N + 1 - 2|M| \cos \theta) \tau \right] \quad (4.23)$$

At zero delay ($\tau = 0$), the correlation starts well below unity, showing strong antibunching of the emitted photons. As τ increases, $g^{(2)}(\tau)$ recovers to unity, indicating that at long delays the photon statistics become Poissonian. The solid blue curve corresponds to the squeezed vacuum case ($|M| > 0$), while the dashed red curve corresponds to the thermal case without squeezing ($M = 0$). Both converge to the same asymptotic limit, but squeezing alters the short-time decay dynamics.

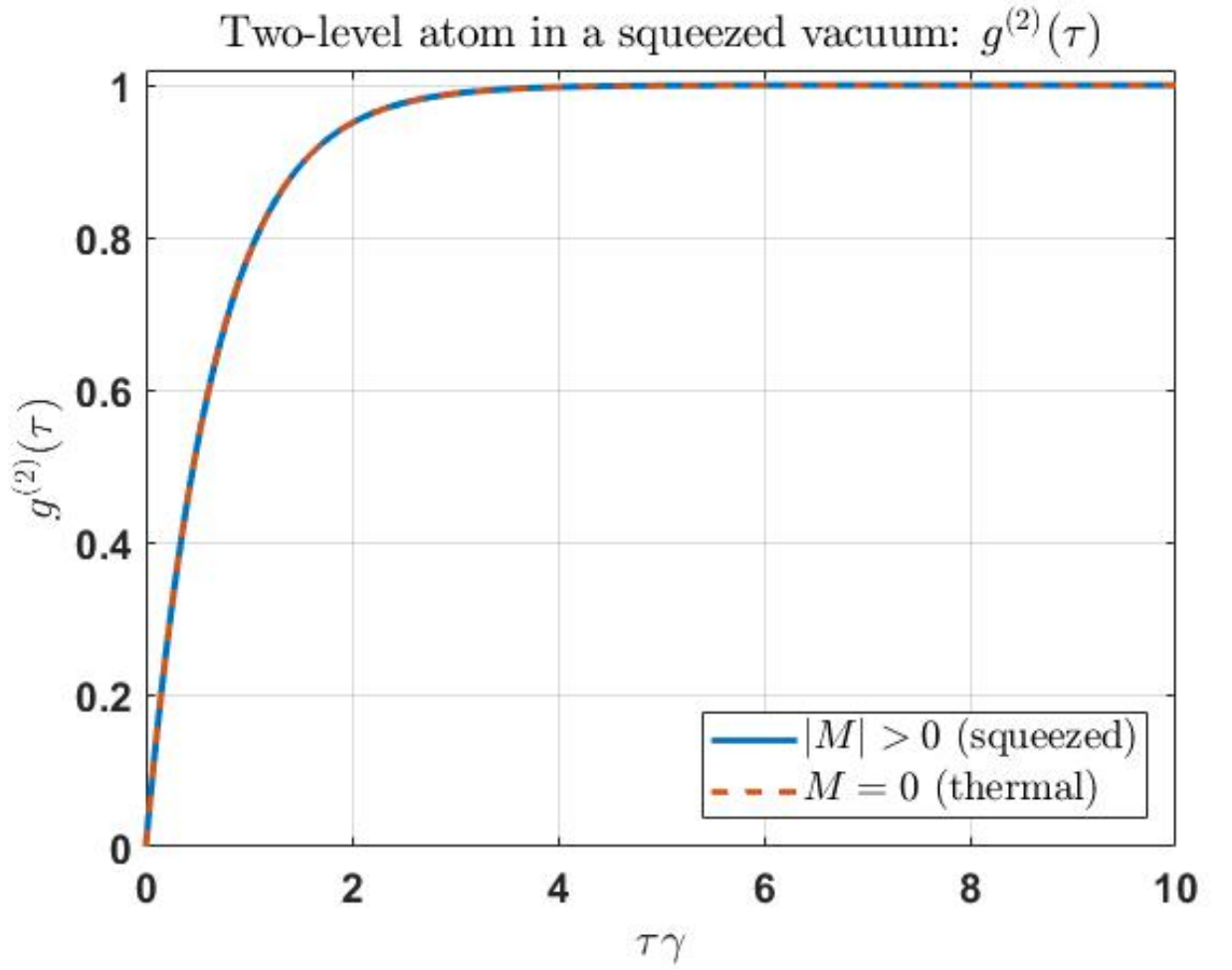


Figure 4.8: Second-order correlation function $g^{(2)}(\tau)$ of a two-level atom in a squeezed vacuum reservoir.

Chapter 5

CONCLUSION AND RECOMMENDATION

5.1 Conclusion

In this thesis, the light emission characteristics of a two-level atom interacting with a squeezed vacuum field have been thoroughly investigated both analytically and numerically. The study focused on understanding how quantum squeezing modifies spontaneous emission, spectral behavior, atomic coherence, and entanglement properties. Using the squeezed Jaynes-Cummings Hamiltonian and the master equation formalism, we derived the time evolution of atomic population and coherence under the influence of a phase-sensitive reservoir.

The simulation results revealed that the presence of a squeezed vacuum enhances the effective Rabi frequency, leading to faster population oscillations compared to the ordinary vacuum case. The emission spectra, calculated through the quantum regression theorem, exhibit Lorentzian profiles whose linewidths are narrowed as the squeezing parameter increases. This subnatural linewidth narrowing demonstrates that squeezed reservoirs can suppress quantum noise in one quadrature while amplifying it in the orthogonal one. Such phase-sensitive modification of emission confirms the nonclassical character of squeezed light. Furthermore, the two-photon correlations ($M(r)$) were shown to grow faster than the mean photon number ($N(r)$) with increasing squeezing, signifying the dominance of pair correlations in squeezed vacuum fields. The study also demonstrated that when two identical two-level atoms are coupled to a common squeezed vacuum reservoir, steady-state entanglement emerges even without direct interaction, confirming the squeezed vacuum as a useful quantum resource. The joint quadrature variances of a two-mode squeezed vacuum decrease exponentially below the vacuum limit, providing additional evidence of noise reduction and quantum correlation between the modes. Overall, the results confirm that the squeezed vacuum

profoundly alters spontaneous emission and coherence dynamics, leading to enhanced quantum correlations, reduced noise, and potential steady-state entanglement. These findings contribute valuable insight into the role of squeezed light in quantum optics and have promising implications for advanced applications in quantum metrology, precision spectroscopy, and quantum information processing. The outcomes of this research affirm that squeezed vacuum fields can be effectively used to engineer quantum states of matter and light beyond classical limits.

5.2 Recommendation

Based on the theoretical investigation of light emission from squeezed vacuum interaction with two-level atoms, the following recommendations are forwarded:-

1. Conduct experiments to verify linewidth narrowing, modified spontaneous emission, and antibunching in cavity QED or trapped-ion systems.
2. Perform numerical simulations using master-equation and Monte-Carlo methods for varied squeezing parameters and coupling strengths.
3. Study multiple atoms and strong-coupling effects to explore superradiance, sub-radiance, and multi-atom entanglement.
4. Analyze the impact of noise, dephasing, and imperfect squeezing, and propose strategies to preserve non-classical behavior.
5. Apply squeezed-reservoir engineering in quantum technologies for entanglement, noise reduction, and precision spectroscopy.

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