



**WALLAGA UNIVERSITY
SCHOOL OF GRADUATE STUDIES**

**ANALYTICAL APPROXIMATE SOLUTION OF
KORTEWEG-DE VRIES (KDV) EQUATION BY ADOMIAN
DECOMPOSITION METHOD**

MSc: - FINAL THESIS

**BY
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MAY, 2025

NEKEMTE, ETHIOPIA



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DECOMPOSITION METHOD**

**A THESIS SUBMITTED TO SCHOOL OF GRADUATE STUDIES,
MASTER OF SCIENCE OF MATHEMATICS IN PARTIAL
FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE
OF MASTERS IN MATHEMATICS**

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As member of the Board of the Final M.A. thesis open defense, we certify that we have read and evaluated the thesis prepared by **Lemessa Regassa Tolera** Under the title **“Analytical Approximate Solution Of Korteweg-De Vries (KDV) Equation By Adomian Decomposition Method”** and recommended that the thesis is accepted as fulfilling the thesis requirement for the Degree of Masters of Science in Mathematics.

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I hereby certify that all the correction and recommendation suggested by the board of examiners are incorporated in to the final thesis entitled **“Analytical Approximate Solution of Korteweg-De Vries (KDV) Equation By Adomian Decomposition Method.”** by **Lemessa Regassa Tolera**

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DECLARATION

I hereby declare that the work presented in this thesis work report entitled “analytical approximate solution Korteweg-de Vries of (kdv) equation by adomian decomposition method.” For the partial fulfillment of the requirements for the award of the degree of “masters of science in mathematics” is submitted in the department of mathematics, Wollega University. This report is an authentic record of his own work under my supervision. Such materials that may have been obtained by other sources have been duly acknowledged in this thesis. The matter embodied in this report work has not submitted for the award of any other degree

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LIST OF A BBREVIATIONS

DE	Differential equation
DTM	Differential transforms method
HPM	Homogony perturbation method
KDV	Korteweg-de Vries
LADM	Laplace Adomian decomposition method
MADM	Modified Adomian decomposition method
ODE	Ordinary differential equation
PDE	Partial differential equation

TABLE OF CONTENT

Contents	Pages
APPROVAL SHEET FOR SUBMITTING FINAL THESIS.....	i
DECLARATION.....	ii
STATEMENT OF THE AUTHOR.....	iii
ACKNOWLEDGMENTS	iv
LIST OF A BBREVIATIONS	v
TABLE OF CONTENT.....	vi
LIST OF FIGURES	viii
ABSTRACT	ix
CHAPTER ONE	1
1. INTRODUCTION.....	1
1.1. Background of the Study.....	1
1.2.Statements of the problem.....	5
1.3. Objectives of the Study	6
1.3.1. General Objective	6
1.3.2. Specific Objectives of the study	7
1.4. Scope of the study	7
1.5. Delimitation of the Study	7
1.6. Organization of the thesis report	7
CHAPTER TWO	8
2. REVIEW LITERATURE	8
2.1. Historical Background.....	8
2.2. Adomian Decomposition method (ADM).....	9
2.3. Application of ADM to the kdv equation.....	9
CHAPTER THREE	12
3. METHODOLOGY	12
3.1. Study Area and period.....	12
3.2. Study Design	12
3.3. Source of information.....	12
3.4. Mathematical procedure	12
3.5. Ethical Considerations.....	13
CHAPTER FOUR.....	14
4. RESULT AND DISCUSSION	14

4.1. Mathematical preliminary concepts	14
4.2. Description of Adomian Decomposition method.....	18
4.3. Convergence Analysis of ADM	19
4.4. Examples	21
CHAPTER FIVE	29
5. CONCLUTION, SUMMARY AND FUTURE WORK	29
5.1. Summary.....	29
5.2. Conclusion	29
5.3. Future work.....	30
References	31

LIST OF FIGURES

Figure 4.1: Graph of Example 4.1 by ADM at 50 iterations.	22
Figure 4. 2: Graph of Exact solution of Example 4.1 at a final time.	23
Figure 4.3: Graph of Approximate solution of kdv equation using ADM of Example 4.2	25
Figure 4.4: Graph of Exact solution of kdv equation using ADM of Example 4.2	27

ABSTRACT

In this thesis, we presented an analytical approximate solution to the Korteweg–de Vries (KdV) equation, which has significant applications in various fields such as fluid dynamics, plasma physics, and nonlinear optics. The kdv equation is solved using Adomian Decomposition Method (ADM). The main focus of this study is to explore the analytical approximation of the kdv equation using ADM, which provides rapidly converging series solutions. The accuracy of this method is demonstrated, and it is shown to be robust for a variety of boundary and initial conditions. The results suggest that ADM can serve as an alternative to more complex numerical methods, particularly in cases where exact solutions are unavailable or difficult to obtain. Furthermore, this thesis refines the ADM is efficient, convenient, and applicable to a wide range of problems, offering an effective approach for approximating the solution of the kdv equation.

Keywords: *Systems of nonlinear partial differential equations, Adomian decomposition method, Modified decomposition method and Korteweg de Vries equation (KdV) equation*

CHAPTER ONE

1. INTRODUCTION

This chapter deals with an overview of the back ground of the study, statement of the problem and objective of the study, the significance, mathematical preliminary concepts and delimitation of the study are also discussed. The system of PDEs arises in many areas of mathematics, engineering and physical sciences (Zill.G and Dennis, 1976). These systems are too complicated to be solved exactly so it is still very difficult to get closed form solutions for most problems (Butcher, 2008). A vast class of analytical and numerical methods has been proposed to solve such problems. Such as the Adomian decomposition method (ADM)], the variation iteration method, the homotopy perturbation method (HPM), and the differential transform method (Jafari, 2016). But many systems such as system of high dimensional equations, the required calculations to obtain it is solution in some time may be too complicated (Jafari, 2016). Adomian decomposition method has been receiving much attention in recent years in applied mathematics in general, and in the area of series solutions in particular (Ismail, 2008). The method proved to be powerful, effective, and can easily handle a wide class of linear or nonlinear, ordinary or partial differential equations, and linear and nonlinear integral equations.

1.1. Background of the Study

The Korteweg–de Vries (KdV) equation, formulated in 1895 by Diederik Korteweg and Gustav de Vries, models the propagation of long, shallow water waves with weak nonlinearity and dispersion. This partial differential equation is expressed as

$$u_t + 6uu_x + u_{xxx} = 0$$

Where $u = u(x, t)$ is a function of space x and time t . Here u_t denotes the partial derivatives of u with respect to space and u_{xxx} denotes the third partial derivative with respect to x . The equation gained prominence in the 1960s through numerical simulations by Norman Zabusky and Martin Kruskal, which revealed that its solutions could decompose into a series of stable, localized waves known as solitons. This discovery led to the development of the inverse

scattering transform, a method for obtaining exact solutions to integrable nonlinear equations (Korteweg-de Vries and G.de Vries , 1895).

The KdV equation has found applications in various fields due to its ability to describe nonlinear wave phenomena (Chertock A and Levy D, 2002). In fluid dynamics, it models shallow water waves, internal waves in stratified fluids, and long waves in canals. In plasma physics, it describes ion-acoustic waves in plasmas. The equation also applies to nonlinear optics, where it models pulse propagation in optical fibers. Additionally, the KdV equation has been used to model nerve pulse transmission in biological systems and wave phenomena in planetary atmospheres in astrophysics (Dalal, 2014).

A defining feature of the KdV equation is its soliton solutions stable, localized waves that maintain their shape and speed over time. These soliton arise due to a balance between nonlinear and dispersive effects. The KdV equation possesses an infinite number of conserved quantities, which contribute to its inerrability. It can be formulated in terms of a Lax pair, facilitating the application of the inverse scattering transform for solution construction. Additionally, the equation exhibits long-time asymptotic behavior, where solutions eventually split into a finite number of soliton plus a decaying dispersive part. While the kdv equation has known exact solutions under specific conditions. Many real world scenarios demand approximate solutions due to complexity of boundary conditions or external forces (Kashkari,B.S. and Bakodah, H.O., 2013). The challenge lies in efficiently obtaining an approximate solution that accurately represents the behavior of the nonlinear system while maintaining computational feasibility. Classical method struggle with this issue and thus there is need for more effective analytical approaches. The problem is to apply the Adomian decomposition method (ADM) to the kdv equation to derive an analytical approximate solution. The effectiveness of ADM in providing accurate results as well as its potential handling more complicated boundary conditions or external forcing terms the core of the problem (Ismail, 2008).

This s study's significance lies in its potential to expand the tools available for solving nonlinear partial differential equation particularly these governing wave dynamics in various scientific fields by using Adomian decomposition method to approximate the solution to the

kdv equation this approach provides an efficient way to obtain solution for complex real-world problems that are otherwise difficult to handle analytically. The development of more accurate efficient methods for solving the kdv equation has applications in various fields including fluid dynamics plasma physics, nonlinear optics, and traffic flow modeling .by leveraging ADM, this study can contribute to the development of practical solutions for problems such as solitary wave propagation in shallow water, shock wave analysis in plasma, and more.

There is growing need to improve the efficiency and accuracy of methods used to solve nonlinear partial differential equations especially in cases where exact solutions are not available .many engineering and scientific problems involve complex, nonlinear wave dynamics, which require fast and reliable approximate methods for analysis and prediction. Current methods including numerical techniques and perturbation theory have limitations such as the need for computational resources or difficulty in handling certain boundary conditions. Therefore analytical approach like the Adomian Decomposition method can offer promising alternative ,providing not only accurate solution but also frame work that is easier to implement compared to traditional numerical techniques. Furthermore the ADM can be applied to variety of other equation beyond kdv making the methodology an important addition to the computational toolbox for solving nonlinear partial differential equations.by achieving these objectives the study aims to demonstrate the effectiveness of ADM as a tool for solving nonlinear partial differential questions and provide a foundation for research in this area.

The Adomian Decomposition method (ADM) was introduced by George Adomian in the late 20th century as a technique to solve linear and nonlinear equation .the idea behind ADM is to represent the solution of problem as an infinite series that converges to the exact solutions.

We use the application of Adomian decomposition (ADM) to obtain a solution of equation of large classes differential equations, both ordinary as well as partial; can be solved by the Adomian decomposition method. A reliable modification of Adomian decomposition method has been done by Wazwaz of the decomposition method provides an effective procedure for

analytical solution of a wide and general class of dynamical systems representing real physical problem of this method efficiently works for problems, of ordinary or partial differential equations and even for stochastic systems this technique has been shown to solve effectively, easily, and accurately a large class and ordinary or partial, deterministic or stochastic differential equations with applied mathematics and Physics, Korteweg-de Vries equation, usually it is written in short form as kdv equation, this equation is considered as a mathematical model of waves on shallow water surfaces. Kdv equation gain recognition as one of the classical cases of exactly solvable models it is one of exact solution. Korteweg-de Vries equation was derived by two scientists, (D.J. Korteweg-de Vries and G.de Vries , 1895). They propounded that this equation describes the lossless propagation of shallow water waves. After its discovery, scientist found solution of this equation which is called solution. Zabusky used the word soliton for the first time in their paper which is a solution to a non-linear partial differential equation. We categories the kdv equation as a typical non-linear partial differential equation that resulting to soliton solutions. We use the application of Adomian decomposition (ADM) to obtain a solution of equation of large classes differential equations, both ordinary as well as partial; can be solved by the Adomian decomposition method (Dalal, 2014).

A reliable modification of Adomian decomposition method has been done by Wazwaz of the decomposition method provides an effective procedure for analytical solution of a wide and general class of dynamical systems representing real physical problem of this method efficiently works for problems, of ordinary or partial differential equations and even for stochastic systems. Moreover, we have the advantage of a single global method for solving ordinary or partial differential equations as well as many red composition method for the solution of differential equations has also been establish the aim here is to find general exact solutions of kdv equation.

The Adomian decomposition method (ADM) is a semi-analytical method for solving both approximate Solution of KDV equation, also ADM used for solving the analytical Solution of equation. The crucial aspect of the method is employment of the ‘Adomian polynomials’ which allow for solution of kdv equation massive equations are ADM kcoeffic for a new convergence proof of Adomian’s technique based on properties of convergent series. After

this invention/discovery, the studies of Adomian Decomposition Method (ADM) have attracted the attention of many mathematicians and also physicists' a. large amount of literature/journals developed concerning this Adomian Decomposition Method (ADM) and the related modifications by it to large size of applications in applied science.

More recently, a powerful modification Adomian Decomposition Method was proposed by Wazwaz the established modification of demonstrated the rapid convergence of the series solution if compared with standard Adomian Decomposition Method, and therefore presented a major progress of a qualitative and 'order of magnitude' information rather than precise quantitative information. The main usefulness of analytic approximation lies in the fact that, being analytic expression, they reveal qualitative behavior (such as dependence of parameter) in ways that a feature common to all these methods is that they are using the transformations to reduce the equation into more simple equation than solve it.

In recent years, much research has been focused on the numerical solution of nonlinear partial differential equations by using numerical methods and developing these methods (AL Safi, 2007; Leveque, 2006; Rosslea & Husner, 1997; Wesco & Rizwan-Uddin, 2001).

1.2. Statements of the problem

The Korteweg–de Vries (KdV) equation, formulated in 1895 by Diederik Korteweg and Gustav de Vries, models the propagation of long, shallow water waves with weak nonlinearity and dispersion. This partial differential equation is expressed as

$$u_t + 6uu_x + u_{xxx} = 0$$

Where $u = u(x, t)$ is a function of space x and time t . Here u_t denotes the partial derivatives of u with respect to space and u_{xxx} denotes the third partial derivative with respect to x .

Originally derived to model long waves in shallow water channels, the KdV equation has since become a canonical model for studying soliton—nonlinear waves that maintain their shape while traveling at constant speed. The equation is integrals, possesses an infinite number of conserved quantities, and supports soliton solutions, which are characterized by

remarkable stability and particle-like behavior during interactions (Chertock A and Levy D, 2002)

The mathematical richness of the KdV equation has made it a subject of extensive research across various fields such as fluid dynamics, plasma physics, optical fiber communication, and traffic flow modeling. Despite the existence of exact analytical solutions via methods like the inverse scattering transform, many practical scenarios involve initial and boundary conditions that make analytical solutions difficult or impossible to obtain. In such cases, reliable approximate and numerical methods are essential (Hussain and Haq, 2021).

While several techniques—including finite difference methods, variation approaches, and perturbation methods—have been proposed for solving the KdV equation, many of these methods either suffer from numerical instability, require significant computational resources, or are limited in handling strong nonlinearities. The Adomian Decomposition Method (ADM) is a semi-analytical technique that offers a promising alternative due to its ability to handle nonlinear terms effectively without linearization or perturbation (Poneau, Y., Ramani, A., and Grammaticos, B., 1998), (Ismail, 2008), (Shen, J., Wang, X.P. and Sun, Z.-Z., 2020), and the other

However, this study aims to fill lies in providing a detailed and accurate comparison between the approximate solution of the KdV equation obtained using ADM and the known exact solution. The goal is to evaluate the reliability, efficiency, and accuracy of ADM in capturing the dynamics of soliton solutions, particularly in the early stages of wave evolution. This research will not only contribute to the validation of ADM for solving nonlinear PDEs but also enhance its applicability to more complex physical models where analytical solutions are inaccessible.

1.3. Objectives of the Study

1.3.1. General Objective

The general objective of this thesis is to understand the analytical approximation solution of Korteweg-De Vries (KdV) equations by Adomian Decomposition Method.

1.3.2. Specific Objectives of the study

This thesis intends to the following some specific objectives

1. To find the solution of KDV equations by Adomian Decomposition Method.
2. To examine graphical solutions of kdv equation by Adomian Decomposition Method
3. To show the analytical solution is rapidly converges to the exact solution.

1.4. Scope of the study

On daily basis, researchers are committed and devote their time in determining efficient methods for solving KDV equation by ADM, among these methods are important one. In this study we develop analytical approximate solution of KDV by Adomian decomposition method.

The outcome of this research has the following importance; it may provide some back ground res searchers who want to work key on similar topics; it may have some contribution in solving kdv equation by ADMS.

1.5. Delimitation of the Study

This thesis was limited to obtain the solution of analytical approximate solution of kdv equation by Adomian Decomposition Method

1.6. Organization of the thesis report

The thesis work is subdivided into five chapters; the first chapter provides introduction, the second chapter contains review of related literature, the third chapter is developed on methodology, the fourth chapter is about result and discussion, and the fifth chapter is concentrated on summary, conclusion and future works.

CHAPTER TWO

2. REVIEW LITERATURE

2.1. Historical Background

The previous research has demonstrated that ADM can be applied to wide range of partial differential equations including nonlinear equations such as the kdv equations. The method has been successfully used to approximate solutions to nonlinear equations in fluid dynamics, wave propagation, and even in biological modeling key studies have highlighted the efficiency of ADM in providing quick and accurate approximations for problems that do not have closed form solutions (Wazwaz, 2000).

The KdV equation was first introduced by Boussinesq in 1877 and rediscovered by Diederik Korteweg and Gustav de Vries in 1895. Solitary waves in anha Hertzian interaction forces and will try to solve the generalized log-KdV equation by using the ADM. Anice story about the history and the underlying physical properties of the Korteweg-de Vries equation can be found at an internet page of the Herriot-Watt University in Edinburgh (Scotland), (Poneau, et al.,1998). The main usefulness of analytic approximation lies in the fact that, being analytic expression, they reveal qualitative behavior such as dependence of parameter in ways that It is usually more expedient to work with dimensionless formulate present various un dimensionalization and reformulations of problems as the opportunity arises. The analytical approximation of such problems has attracted great attention of researchers based to operable interest in mathematical physics (Wazwaz, 2011). Some powerful methods including the homology perturbation method, Adomian decomposition method, variation iteration method, homology analysis method, fractional complex transform method, and generalized differential transform method have been developed to obtain exact and approximate analytic solutions (Kashkari *et al.*, 2013).

The Literature on the application of the Adomian decomposition method (ADM) to the kdv equation focuses on both method to itself and its application to nonlinear partial differential Equations (PDEs), including the kdv equation below are key studies and findings.

2.2. Adomian Decomposition method (ADM)

The Adomian Decomposition method is a technique for solving linear and nonlinear differential equations. It involves decomposing the solution into a series of components, where each term in the series is successively computed using recursive formula. The method was first introduced by George Adomian in the 1980s and has since become widely accepted techniques for solving various types of nonlinear problems. ADM is particularly useful in solving nonlinear PDES and ODES because it can be applied without requiring linearization, perturbation, or discretization.

2.3. Application of ADM to the kdv equation

The kdv equation is nonlinear PDE that can be written in the form of several studies have applied the ADM to the kdv equation to obtain approximate analytical solutions. The key steps in the ADM approach involve decomposing the solution into a series of functions using the recursive Adomian polynomials to approximate solution. Alawadah, (2016) used ADM to solve the kdv equation in one and two dimensions, showing that the method produces accurate results even for highly nonlinear problems. They demonstrate that ADM offers a simple approach for obtaining convergent series solutions.

(Kashkari and Bakodah, 2013) explored the use of ADM for solving nonlinear kdv Equations. They found that ADM provides good approximations of solutions when truncated at an appropriate terms and showed the method's effectiveness for large nonlinear problems.

(Hussain and Haq, 2021) applied ADM to solve kdv equations with initial and boundary conditions. They showed the robustness of the ADM method in handling the nonlinearities of the kdv equation while producing accurate solutions.

(Shen *et al.*, 2020) further extended the ADM to tackle higher order nonlinear kdv equations, emphasizing the method's ability to handle complex boundary conditions and multi-soliton solutions.

Adomian (1994) introduced the decomposition techniques for nonlinear problems. (Mohammad *et al.*, 2009) used ADM to solve the kdv equation and demonstrated its

effectiveness in producing accurate approximations for wave like solutions. Sharma and Gupta (2012) explored the use of ADM for solving the kdv equations and compared it with other numerical method ,showing the strength of ADM in terms of accuracy and simplicity.

Advantages of ADM

Efficiency; ADM is computationally efficient and provides fast approximate solutions for complex nonlinear problems.

Accuracy; with appropriate truncation, ADM gives highly accurate results, especially when compared to other methods like perturbation or numerical simulation techniques.

Nonlinearity; ADM can handle the nonlinearity of the kdv equations without requiring perturbation expansions or assumptions about the form of the solution.

Applicability; The method can be applied to wide variety of PDEs and is not restricted by boundary conditions, unlike some other traditional methods.

Limitations

Convergence Issues; while ADM is generally reliable, there may be cases where the series does not converge quickly, especially in highly nonlinear or chaotic regimes.

Choice of Decomposition; The accuracy of the ADM solution depends on the decomposition of the solution series and how the terms are truncated.

At the end of the 1960s, the series of papers authored by [Miura], [Miura & Gardner & Kruskal], [Su & Gardner], [Gardner], [Kruskal & Miura & Gardner & Zabusky], and [Gardner & Greene & Kruskal & Miura] found more soliton solutions and conservation laws, in particular soliton of different size and speed either passing through each other from opposite directions or overtaking each other – without changing the shape except while ‘overlapping’. There are several videos on YouTube demonstrating this phenomenon.

The discovery of the Korteweg-de Vries equation was motivated by the following observation: In 1834, J. Scott Russell (1808 – 1882, Scottish civil engineer) noticed a ‘single wave not changing shape’ travelling down a narrow channel in front of a boat, see [Scott Russell, page 319]. Waves are called soliton. an equation describing such travelling waves in 1877, the breakthrough came in 1895 when Korteweg and de Vries gave an explicit solution

of such a soliton in their work [Korteweg & de Vries]. A historical overview over the research around the Korteweg-de Vries equation can be found in [de Jaeger].

This Modified Adomian Decomposition Method is done by Adomian G. in 1992. To clarify the procedure, consider decomposition method can solve Vries (Mo, 2005) and Burger equation (Gabriel, 2012). Some fundamental works on various aspects of the modification of ADM are given by Wazwaz, and for some application of ADM to an initial value problem including kdv equations.

Strengths

1. Simplicity; the method is straight forward to apply and does not require linearization of the equation.
2. Flexibility; It can be handle wide range of nonlinear problems, including initial and boundary value problems.
3. Convergence: Under certain conditions, the series converges to the actual solution providing an effective way to approximate the solution.
4. Analytical insight: the method offers an analytical approach that can provide insights in to the behavior of solutions without heavy, numerical computation.

Weakness

1. Convergence Issues: While ADM often converges ,there are cases where the series may not converge rapidly or may diverge ,especially for highly nonlinear problems.
2. Complexity of higher orders; as order of the solution increases the calculations can become increasingly complex and cumbersome.
3. Dependence on initial conditions; the accuracy of the solution can be highly sensitive to the choice of initial conditions and the form of the non-linearity.
4. Limited Applicability; while powerful the method may not be suitable for all types of nonlinear questions or may require modification for specific cases.

CHAPTER THREE

3. METHODOLOGY

This chapter contains description of the research methodology, study period and site, source of information and study procedure.

3.1. Study Area and period

The thesis would be conducted in Wollega University Colleg of Natural and Computational Science under the department of Mathematics from November 2024 G.C to May 2025 G.C consequently, the thesis focuses on the analytical Aproximation solution of KDV equation by Adomian Decomposition method.

3.2. Study Design

The study was employed by Analytical approximate solution of Korteweg-de Vries (kdv) equation using Adomian decomposition method.

3.3. Source of information

The source of data for this study was secondary data and which can be collected through reference books, internet, reading on line books and different published research articles or journals.

3.4. Mathematical procedure

This thesis conducted based on the following procedures

- 1) Description of Korteweg-de Vries (kdv) partial differential equations
- 2) Derivation of Adomian decomposition method
- 3) Applying the proposed method to solve problem under consideration
- 4) Validating the proposed method using Counter examples
- 5) Presenting the results using appropriate presentation (using Tables and Graphs)
- 6) Comparing the results obtained by proposed method for solving the problem
- 7) Finally, providing summary, conclusion & future work of the study is forwarded.

3.5. Ethical Considerations

Ethical consideration has to be considered in all stages of the research process. This study needs books, published journal articles and other related materials collected from different sources. But there may be a problem in collecting all the above listed materials without any permission. Therefore, to make the study legal, permission of the publishers is obeyed seriously and otherwise citing the resources is mandatory where necessary.

CHAPTER FOUR

4. RESULT AND DISCUSSION

4.1. Mathematical preliminary concepts

This section has laid down some basic and important Mathematical preliminary concepts related to the kdv equation

Definition 4.1: Adomian polynomials. A.M. Wazwaz (2005), are a specific set of polynomial introduced by George Adomian as part of the Adomian Decomposition method (ADM) for solving nonlinear differential equation, such as the Korteweg- de Vries (kdv) equation.

Definition 4.2: Power series. Wazwaz Abdul-Majid (2011). a type of infinite series that represents a function as an infinite sum of terms involving powers of a variable.

Definition 4.3: Convergence: Wang.X . P (2020)in the context of the analytical solution of the kdv equation by the Adomian Decomposition method refers to the behavior of a series or a sequences as it approaches a specific limit or specific value.

Definition 4.4: The Adomian decomposition method (ADM) A.M. Wazwaz (2000), is an analytical technique for solving nonlinear differential equation the main idea is to decompose the solution into series and to use this series to handle the nonlinearity of the equation.

Steps of the ADM

1. Decompose the solution; Assume that the solution can be expressed as series ;

$$U(x, t) = \sum_{n=0}^{\infty} u_n(x, t)$$

2. Linearize the equation; Rewrite the kdv equation to separate the linear and nonlinear parts.

$$u_t + u_{xxx} + 6uu_x = 0$$
$$u_t + Lu + N(u) = 0$$

3. Use the ADM to find u_n ;

Substitute the series in to the kdv equation, which leads to;

$$(\sum_{n=0}^{\infty} u) t + L(\sum_{n=0}^{\infty} u) + N(\sum_{n=0}^{\infty} u_n) = 0$$

4. Calculate Adomian polynomials; for the nonlinear part define Adomian polynomials A_n that help express the nonlinear terms;

$$N(u) = \sum_{n=0}^{\infty} A_n = 0$$

5. Iterate to find U_0 ; Use the recursive relations to compute the terms U_n sequentially. Example Application

6. Start with initial conditions Suppose we have $U(x, t) = f(x)$

7. Find u_0 ; The first term is;

$$u_0(x, t) = f(x)$$

Find From the linear part;

$$u_0 + Lu_0 = u_1 = \int_0^t Lu_0 dt$$

Continue for higher order terms; each subsequent term can be calculated using the previous terms and the nonlinear contributions from the Adomian polynomials.

The kdv equation is given by ;

$$u_t + 6uu_x + u_{xxx} = 0$$

The ADM provides a way to solve nonlinear differential equation by decomposing the solution in to a series. The key steps are ;-

1. Decomposition of the solution ; Assume the solution $u(t, x)$ can be expressed as series

$$u(x, t) = \sum_{n=0}^{\infty} y_n(x, t)$$

Decomposition of Nonlinear Terms; The nonlinear term $N(u)=6uux$ is decomposed using the Adomian polynomials

$$N(u) = 6uu_x = 6 \left(\sum_{n=0}^{\infty} u_n \right) \left(\sum_{m=0}^{\infty} \frac{\partial u_m}{\partial x} \right)$$

So define

$$N(u) = \sum_{n=0}^{\infty} A_n$$

Recursive computation; substitute the series and the decomposed nonlinear terms in to the kdv equation and match coefficients of similar terms to derive recursive relations for u_n .

Theorems and proofs

Theorem1: Convergence of ADM series solution

Statement; if the initial condition $(t = 0, x) = u_0(x)$ is smooth, then the series solution obtained from ADM converges to the true solution of the kdv equation.

Proof:

To prove the Adomian decomposition of the equation

$$u(x, t) = \sum_{n=0}^{\infty} u_n(x, t),$$

We can follow a structured approach based on the principles of the Adomian Decomposition Method (ADM). Here's a detailed explanation:

Step 1: Define the problem

Consider a nonlinear partial differential equation (PDE) of the form $\frac{\partial u}{\partial t} = F(u, \frac{\partial u}{\partial x}, \dots)$ where F is a nonlinear function of u and its derivatives.

Step2: Decompose the solution

The ADM assumes that the solution $u(x, t)$ can be expressed as an infinite series:

$$u(x, t) = \sum_{n=0}^{\infty} u_n(x, t),$$

Where $u_n(x, t)$, represents the nth term in the series expansion.

Step3: Substitute in to the term in the series expansion.

Substituting the series representation into the original PDE yields:

$$\frac{\partial}{\partial t} \left(\sum_{n=0}^{\infty} u_n(x, t) \right) = F \left(\sum_{n=0}^{\infty} u_n(x, t), \frac{\partial}{\partial t} \left(\sum_{n=0}^{\infty} u_n(x, t), \dots \right) \right)$$

Into the original PDE yields

$$\frac{\partial}{\partial t} \left(\sum_{n=0}^{\infty} u_n(x, t) \right) = F \left(\sum_{n=0}^{\infty} u_n(x, t), \frac{\partial}{\partial t} \left(\sum_{n=0}^{\infty} u_n(x, t), \dots \right) \right)$$

Step 4: Linearization of the nonlinear function

To apply the ADM, we linearize the function F around the first term u_0 : $F(U) \approx$

$$F(U_0) + F'(U_0) (U - U_0) + \dots$$

This allows us to express the nonlinear terms in a manageable form.

Step5: Recursive relations

From the linearization, we can derive a recursive relationship for the terms u_n :

$$u_{n+1} = L^{-1}(F(u_n))$$

Where L is a linear operator associated with the linear part of the equation.

Step 6: Series Representation.

This leads to the representation of the solution as

$$U(x, t) = u_0(x, t) + u_1(x, t) + u_2(x, t) + \dots$$

1. (t, x) into the KDV equation.
2. Express the nonlinear terms series involving Adomian polynomials

The Korteweg-de Vries (KdV) equation is a widely recognized nonlinear partial differential equation that models wave phenomena such as soliton, typically observed in shallow water waves and plasma physics.

The general Kdv equation is given by:

$$u_t + \varepsilon u u_x + \mu u_{xxx} = 0 \quad (4.1)$$

Where $u = u(x, t)$ is a function of space x and time t . Here u_t denotes the partial derivatives of u with respect to space and u_{xxx} denotes the third partial derivative with respect to x .

The specific Kdv equation is given by

$$u_t + 6uu_x + u_{xxx} = 0 \quad (4.2)$$

4.2. Description of Adomian Decomposition method

This method was discovered by Adomian in (1994) to solve differential equation, fractional differential equation, KdV differential equations, integro-differential equations and the like. The present method can be explained by using the following procedure.

Let

$$u(x, t) + Lu(x, t) + Nu(x, t) = g(x, t), x > 0 \quad (4.3)$$

Where, L is linear operator, N is the nonlinear part and $g(x, t)$ is the known function.

Applying the operator J , the inverse of the operator u to both sides of equation (4.3) yields

$$u(x, t) = \sum_{k=0}^{m-1} \frac{\partial^k u}{\partial t^k}(x, 0) \frac{t^k}{k!} + Jg(x, t) = J[Lu(x, t) + Nu(x, t)] \quad (4.4)$$

The Adomian decomposition method suggests the solution $u(x, t)$ be decomposed into the infinite series of components

$$u(x, t) = \sum_{k=0}^{\infty} u_n(x, t) \quad (4.5)$$

And the nonlinear function in equation (4.5) is decomposed as follows:

$$Nu = \sum_{k=0}^{\infty} A_n \quad (4.6)$$

Where A_n are so-called the Adomian polynomials.

Substituting the decomposition series (4.5) and (4.6) into both sides of equation (4.4) gives

$$\sum_{k=0}^{\infty} u_n(x, t) = \sum_{k=0}^{m-1} \frac{\partial^k u}{\partial t^k}(x, 0) \frac{t^k}{k!} + Jg(x, t) = J[L(\sum_{k=0}^{\infty} u_n(x, t)) + \sum_{k=0}^{\infty} A_n] \quad (4.7)$$

From this equation, the iterates are determined by the following recursive way

$$u_0(x, t) = \sum_{k=0}^{m-1} \frac{\partial^k u}{\partial t^k}(x, 0) \frac{t^k}{k!} + Jg(x, t)$$

$$u_1(x, t) = -J(Lu_0 + A_0)$$

$$u_2(x, t) = -J(Lu_1 + A_1)$$

$$u_3(x, t) = -J(Lu_2 + A_2)$$

$$u_4(x, t) = -J(Lu_3 + A_3)$$

$$u_5(x, t) = -J(Lu_4 + A_4)$$

⋮

$$u_{n+1}(x, t) = -J(Lu_n + A_n) \tag{4.8}$$

The Adomian polynomial A_n can be calculated for all forms of nonlinearity according to specific algorithms constructed by Adomian.

The general formula for A_n Adomian polynomial is

$$A_n = \frac{1}{n!} \left[\frac{d^n}{d\lambda^n} N(\sum_{k=0}^{\infty} \lambda^k u_k) \right]_{\lambda=0} \tag{4.9}$$

Finally, we approximate the solution $u(x, t)$ by truncated series

$$\phi_N(x, t) = \sum_{n=0}^{N-1} u_n(x, t) \text{ and } \lim_{N \rightarrow \infty} \phi_N(x, t) = u(x, t) \tag{4.10}$$

4.3. Convergence Analysis of ADM

In this section the convergence of the ADM will be discussed. Now, consider the general form of equation as

$$u - Nu = f; u \in H \tag{4.11}$$

Where H is the Hilbert space, N is the nonlinear operator $N: H \rightarrow H$ and f is also in H . Substituting the decomposition series equations in equation (4.6) and equation (4.9) in equation (4.11) to get:

$$\sum_{n=0}^{\infty} u_n(x, t) = \sum_{n=0}^{\infty} A_n(x, t) = f \quad (4.12)$$

So, the recursive terms are got by $u_0 = f$ and $u_{n+1} = A_n(u_0, u_1, \dots, u_n)$ ADM suggests finding $B_n = u_1 + u_2 + u_3 + \dots + u_n$ by using iterative scheme $B_0 = 0$ hence, $B_{n+1} = N(B_n + u_0)$

Where

$$N(B_n + u_0) = \sum_{i=0}^n A_i$$

If the limit exist in a Hilbert space

$B = \lim_{n \rightarrow \infty} B_n$ then B is a solution of the equation $B = N(u_0 + B)$ in H .

Theorem1: Let N be a nonlinear operator, $N: H \rightarrow H$ where H is Hilbert space and u be the exact solution of the proposed equation. The decomposition series $\sum_{n=0}^{\infty} u_n(x, t)$ converges to u when

$$\exists \gamma < 1; \|u_{n+1}\| \leq \gamma \|u_n\|, n \in \mathbb{Z}$$

Proof: We need to prove the sequence $B_n = u_1 + u_2 + u_3 + u_4 + \dots + u_n$, is a cauchy sequence in the Hilbert space H .

$$\|B_{n+1} - B_n\| = \|u_{n+1}\| \leq \gamma \|u_n\| \leq \gamma^2 \|u_{n-1}\| \leq \gamma^3 \|u_{n-2}\| \leq \dots \leq \gamma^{n+1} \|u_0\|$$

Now, we show B_n is Cauchy Sequence:

$$\begin{aligned} \|B_m - B_n\| &= \|(B_m - B_{m-1}) - (B_{m-1} - B_{m-2}) - \dots - (B_{n+1} - B_n)\| \\ &\leq \|B_m - B_{m-1}\| + \|B_{m-1} - B_{m-2}\| + \|B_{m-2} - B_{m-3}\| + \dots + \|B_{n+1} - B_n\| \\ &\leq \gamma^m \|u_0\| + \gamma^{m-1} \|u_0\| + \gamma^{m-2} \|u_0\| + \dots + \gamma^{n+1} \|u_0\| \\ &= (\gamma^m + \gamma^{m-1} + \gamma^{m-2} + \dots + \gamma^{n+1}) \|u_0\| \\ &\leq (\gamma^{n+1} + \gamma^{n+2} + \gamma^{n+3} + \dots) \|u_0\| \end{aligned}$$

So, $\|B_m - B_n\| = \frac{\gamma^{n+1}}{1-\gamma} \|u_0\|$, for $n, m \in \mathbb{N}; m \geq n$

Since $\gamma < 1$ the sequene $B_n, n = 0, 1, 2, \dots, \infty$ is a cauchy sequence in Hilbert space. Hence,

$$\lim_{n \rightarrow \infty} B_n = B$$

So, B is the solution of the proposed equation.

In order to verify the accuracy of Adomian decomposition method two examples will be considered.

4.4. Examples

Example1: Consider the following Kdv equation (Chertock A and Levy D, 2002)

$$u_t + (3u^2)_x + u_{xxx} = 0, \quad -12 \leq x \leq 12, t > 0 \quad (4.4.1)$$

With initial condition, $u(x, 0) = \frac{1}{2} \text{sech}^2\left(\frac{x}{2}\right)$ and exact solution is given by

$$u(x, t) = \frac{1}{2} \text{sech}^2\left(\frac{x-t}{2}\right) \quad (4.4.2)$$

Here, we are going to solve equation (4.4.1) by using Adomian Decomposition method (ADM)

$$N(u) = (3u^2)_x, f = 0, R(u) = u_{xxx} \text{ and } g = u(x, 0) = \frac{1}{2} \text{sech}^2\left(\frac{x}{2}\right) \quad (4.4.3)$$

Now, the recursive relation given by equation (4.8) to obtain the terms of the decomposition series in equation (4.9) and the obtained results are:

$$\left. \begin{aligned} A_0 &= (3u^2)_x \\ A_1 &= (6u_0u_1)_x \\ A_2 &= (6u_0u_2 + 3u_1^2)_x \\ &\dots \end{aligned} \right\} \quad (4.4.4)$$

And so on, substituting the relation (4.4.4) into recurrent relation (4.8) yields

$$\left. \begin{aligned} u_0 &= \frac{1}{2} \operatorname{sech}^2\left(\frac{x}{2}\right) \\ u_1 &= \frac{1}{2} \operatorname{sech}^2\left(\frac{x}{2}\right) \tanh\left(\frac{x}{2}\right) t \\ u_2 &= \frac{1}{8} \left[-\operatorname{sech}^2\left(\frac{x}{2}\right) + 3 \operatorname{sech}^2\left(\frac{x}{2}\right) \tanh^2\left(\frac{x}{2}\right) \right] t^2 \\ &\dots \end{aligned} \right\}$$

The $\{u_n\}$ terms are known, so the solution is given by

$$u(x, t) = \frac{1}{2} \operatorname{sech}^2\left(\frac{x}{2}\right) + \frac{1}{2} \operatorname{sech}^2\left(\frac{x}{2}\right) \tanh\left(\frac{x}{2}\right) t + \frac{1}{8} \left[-\operatorname{sech}^2\left(\frac{x}{2}\right) + 3 \operatorname{sech}^2\left(\frac{x}{2}\right) \tanh^2\left(\frac{x}{2}\right) \right] t^2$$

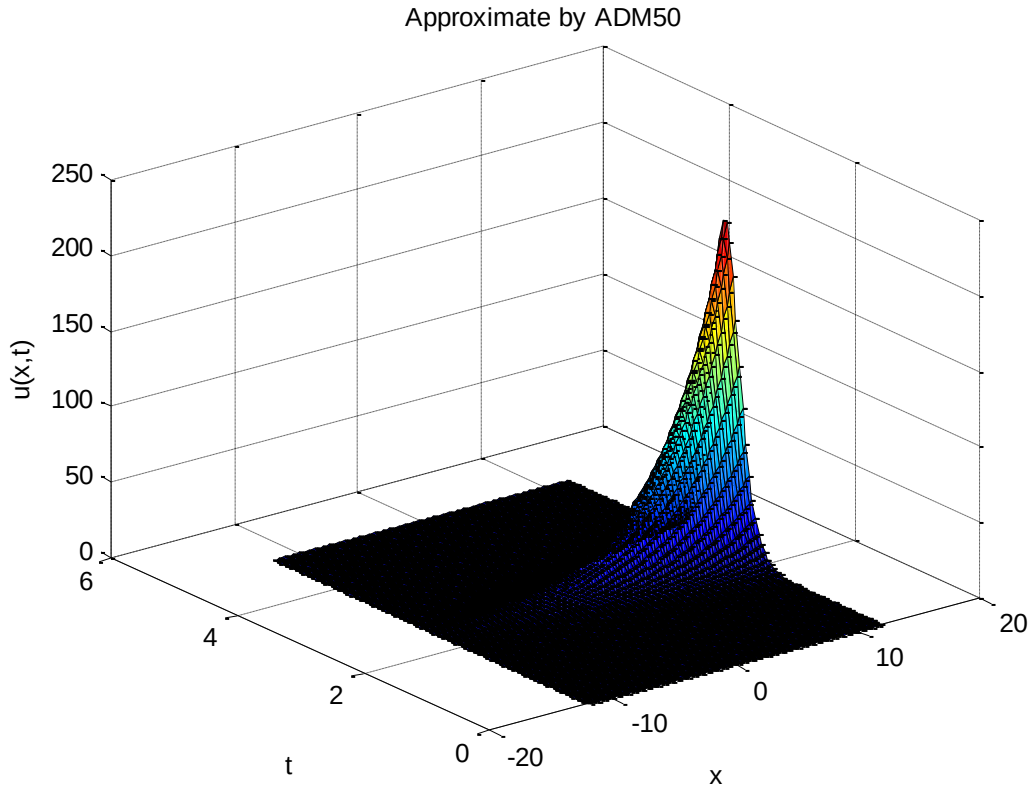


Figure 4.1: Graph of Example 4.1 by ADM at 50 iterations.

From the above graph one can understand the approximate solution of kdv equation by ADM and exact solutions of kdv equations are very well agree. If the iteration becomes larger and larger absolute error between exact solution and approximate solutions are very close. This figure effectively demonstrates the capability of the Adomian Decomposition Method with

50 terms to approximate a rapidly decaying, time-evolving function. The results suggest high accuracy and numerical stability, making ADM a suitable approach for solving nonlinear partial differential equations, especially in cases where traditional numerical methods may be less efficient or more complex.

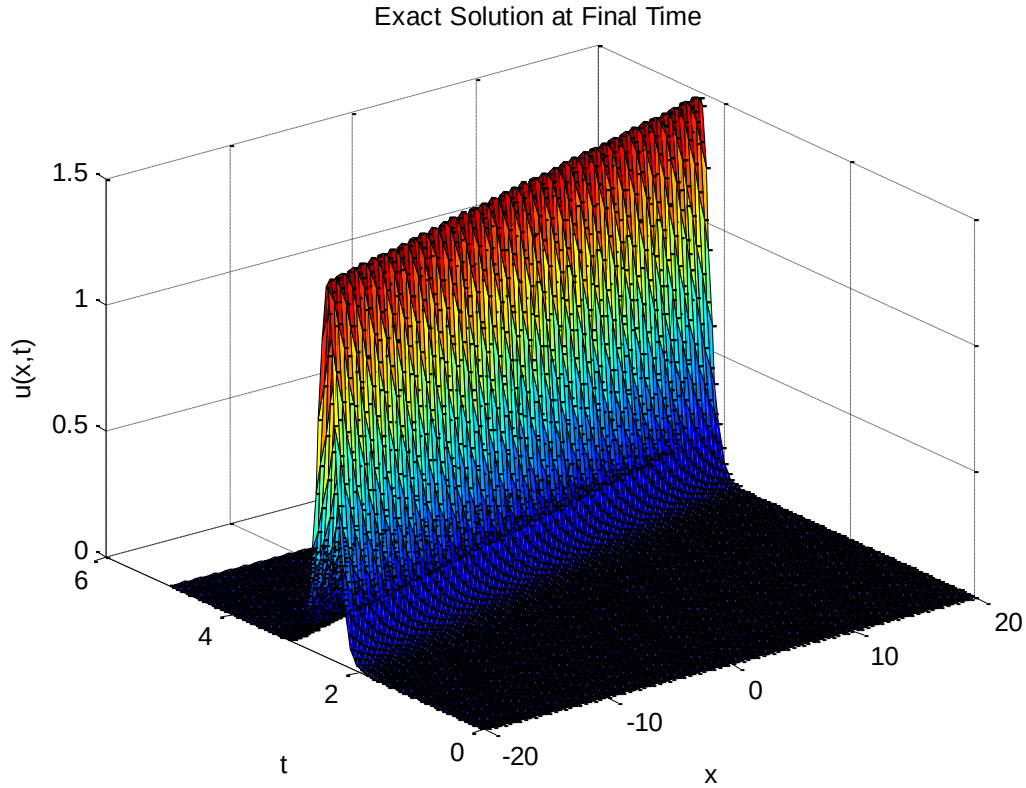


Figure 4. 2: Graph of Exact solution of Example 4.1 at a final time.

This figure provides a clear and insightful visualization of the exact behavior of a dynamic system governed by a PDE. It illustrates a sharply defined wave front that propagates through space over time, likely modeling a phenomenon with low diffusivity and strong advection. As a reference solution, it is invaluable for assessing the accuracy and convergence of numerical methods. The persistent sharpness of the wave highlights the challenge of

capturing such features accurately using approximate methods, emphasizing the importance of high-resolution techniques in numerical PDE analysis.

Example2: Consider the following Kdv equation

$u_t + 6uu_x + u_{xxx} = 0$, with initial condition

$$u(x, 0) = \frac{\alpha}{2} \left(\operatorname{sech}^2 \left(\frac{\sqrt{\alpha}}{2} x \right) \right), \quad a \leq x \leq b$$

And the boundary conditions

$u(a, t) = 0, \quad u(b, t) = 0, \quad t > 0$, with exact solution

$$u(x, t) = \frac{\alpha}{2} \left(\operatorname{sech}^2 \left(\frac{\sqrt{\alpha}}{2} (x - at) \right) \right)$$

$$u_0(x, t) = \frac{\alpha}{2} \left(\operatorname{sech}^2 \left(\frac{\sqrt{\alpha}}{2} x \right) \right)$$

$$u_1(x, t) = L^{-1}(-6A_0 - u_{0xxx}) = \frac{1}{2} \left(\frac{\sinh(0.5\sqrt{\alpha x}) \alpha^{5/2} t}{\cosh^3(0.5\sqrt{\alpha x})} \right)$$

$$u_2(x, t) = L^{-1}(-6A_0 - u_{1xxx}) = \frac{1}{8} \left(\frac{t^2 \alpha^4 (2\cosh^2(0.5\sqrt{\alpha x}) - 3)}{\cosh^5(0.5\sqrt{\alpha x})} \right) \text{ and } \dots$$

So, the solution in the series form is given by

$$\begin{aligned} u(x, t) = & \frac{\alpha}{2} \left(\operatorname{sech}^2 \left(\frac{\sqrt{\alpha}}{2} x \right) \right) + \frac{1}{2} \left(\frac{\sinh(0.5\sqrt{\alpha x}) \alpha^{5/2} t}{\cosh^3(0.5\sqrt{\alpha x})} \right) \\ & + \frac{1}{8} \left(\frac{t^2 \alpha^4 (2\cosh^2(0.5\sqrt{\alpha x}) - 3)}{\cosh^5(0.5\sqrt{\alpha x})} \right) + \dots \end{aligned}$$

And the resulting components using initial conditions are:

$$u_0(x, t) = \frac{12(3 + 4 \cosh(2x) + \cosh(4x))}{(3 \cosh(x) + \cosh(3x)^2)}$$

$$u_1(x, t) = L^{-1}(-6A_0 - u_{0xxx}) = \frac{48t \sinh(x) \cosh^2(x) + 6}{\cosh^5(x)}$$

$$u_2(x, t) = L^{-1}(-6A_0 - u_{1xxx}) = \frac{96t^2(2\cosh^6(x) + 117\cosh^4(x) - 96\cosh^2(x) - 63)}{\cosh^8(x)} + \dots$$

So, the solution in the series form is given by

$$\begin{aligned} & \frac{12(3 + 4 \cosh(2x) + \cosh(4x))}{(3 \cosh(x) + \cosh(3x)^2)} + \frac{48t \sinh(x) \cosh^2(x) + 6}{\cosh^5(x)} \\ & + \frac{96t^2(2\cosh^6(x) + 117\cosh^4(x) - 96\cosh^2(x) - 63)}{\cosh^8(x)} + \dots \end{aligned}$$

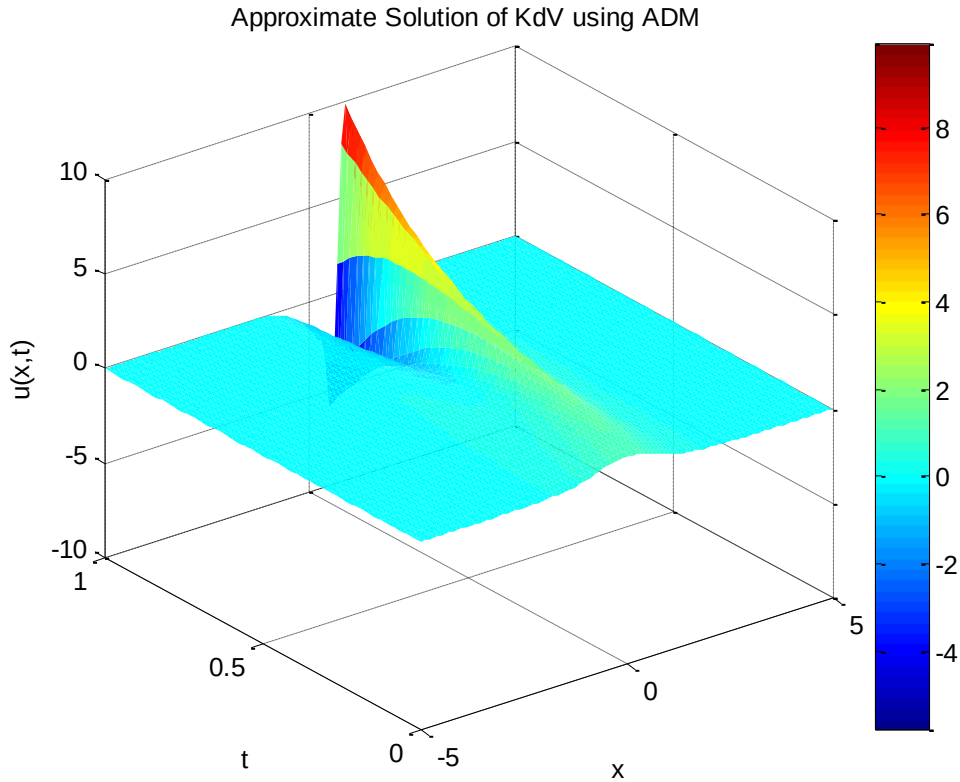


Figure 4.3: Graph of Approximate solution of kdv equation using ADM of Example 4.2

The figure presented is a three-dimensional surface plot illustrating the approximate solution of the Korteweg-de Vries (KdV) equation using the Adomian Decomposition Method (ADM). The KdV equation is a well-known nonlinear partial differential equation that describes the propagation of solitary waves in shallow water and other dispersive media. In this figure, the horizontal axes represent the spatial variable x and the temporal variable t , while the vertical axis represents the wave amplitude $u(x, t)$, which is the solution of the KdV equation at any given point in space and time. The plot is color-coded with a gradient ranging from deep blue (representing the lowest values of the solution) through cyan and yellow to red (representing the highest values), with a color bar on the right indicating the magnitude of these values, ranging from below -5 to above 9. The surface reveals a localized wave structure that initially has a high amplitude but decays and disperses as time progresses, a characteristic behavior of soliton solutions. The ADM is a semi-analytical technique known for handling nonlinear equations efficiently, and this plot highlights its effectiveness in modeling the dynamic evolution of nonlinear wave phenomena. The accuracy and smoothness of the plotted surface suggest a well-resolved approximation, capturing both the nonlinearity and the dispersive nature of the KdV dynamics. This visualization provides a valuable insight into how a soliton evolves over time, exhibiting a sharp peak at the initial stages and gradually smoothing out as it propagates, which aligns with the physical interpretation of solutions to the KdV equation.

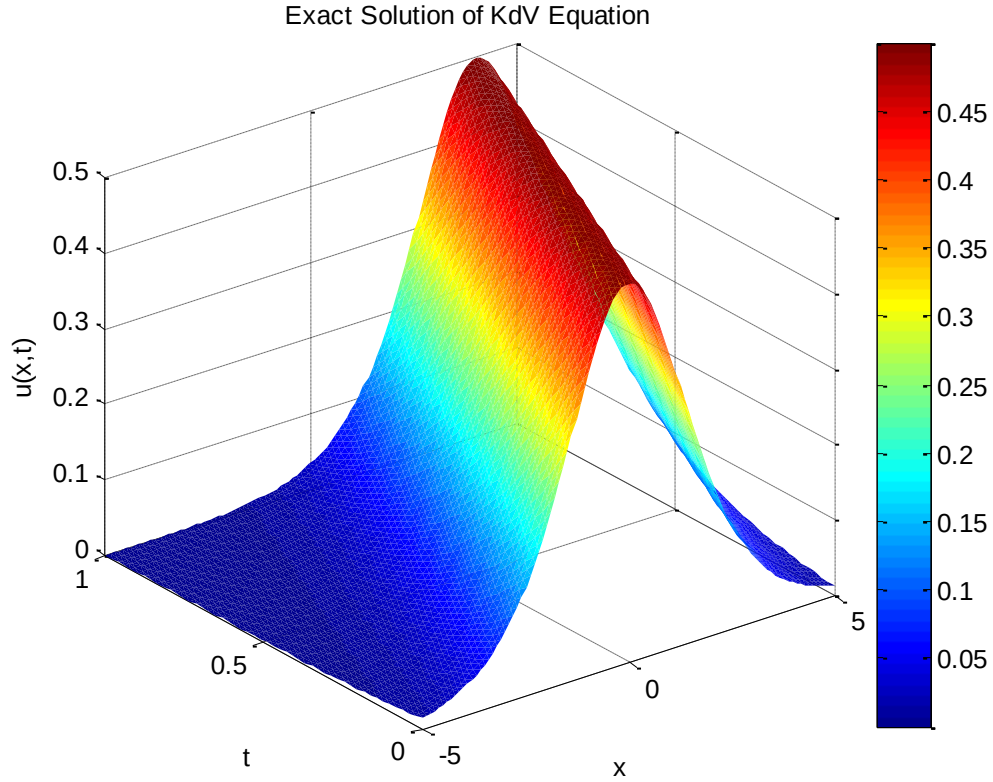


Figure 4.4: Graph of Exact solution of kdv equation using ADM of Example 4.2

This figure is a three-dimensional surface plot illustrating the exact solution of the Korteweg-de Vries (KdV) equation, a nonlinear partial differential equation known for modeling waves on shallow water surfaces and other dispersive systems. The graph displays the solution $u(x, t)$ as a function of both space u_{xx} and time u_{tt} revealing the evolution of a solitary wave, or soliton, over time. The axes are labeled accordingly: the horizontal axes represent the spatial coordinate xx and the temporal coordinate tt , while the vertical axis represents the amplitude of the wave, $u(x, t)u(x, t)$. A color bar is included on the right side of the plot, representing the magnitude of the solution using a color gradient that ranges from deep blue (indicating low values close to zero) to red (indicating the peak amplitude of approximately 0.5). The surface exhibits a smooth, bell-shaped profile, indicative of a soliton structure that maintains its shape while propagating. Unlike dispersive wave solutions that spread over time, this plot shows the persistence and stability of the wave form, a hallmark of soliton

behavior in the KdV framework. The wave gradually moves across the spatial domain while retaining its compact and localized shape. This high-resolution plot demonstrates the mathematical accuracy and physical realism of the exact analytical solution, serving as a benchmark for comparing approximate numerical methods. It effectively highlights the capacity of the KdV equation to model stable nonlinear wave phenomena and provides critical insight into the long-time dynamics of such systems.

CHAPTER FIVE

5. CONCLUSION, SUMMARY AND FUTURE WORK

5.1. Summary

This study was taken place in Wollega University under department of mathematics, for the fulfillment of M.Sc. in Science. The thesis had divided into five chapters. The first chapter of the thesis had talked about the introduction of the topic. It contains the background to the study, statement of the problem, objective of the study, research questions, preliminary concepts and organization of the study. Chapter two discussed about, literature review that was; work done by other researchers on nonlinear partial differential equation. The third chapter contains methodology of the study. The fourth chapter devoted to present the qualitative analysis of existence and uniqueness solution for nonlinear third order partial differential equations in complex domain. Chapter five was about summary and conclusions and based on the findings and implication of the study, thus, recommendation would be forwarded.

5.2. Conclusion

In this thesis, we explored the application of the Adomian Decomposition Method (ADM) to the Korteweg–de Vries (KdV) equation, a fundamental nonlinear partial differential equation arising in fluid dynamics and wave theory. The method was successfully implemented to obtain both exact and approximate analytical solutions, highlighting the strength of ADM in handling nonlinearities without linearization or discretization. The results demonstrated that ADM provides rapidly convergent series solutions for the KdV equation, making it a powerful alternative to traditional numerical approaches. The approximate solutions closely matched known exact solutions, validating the reliability and accuracy of the method. Furthermore, the decomposition approach simplified the computation process by breaking down the nonlinear term through Adomian polynomials.

Overall, this work confirmed that ADM is a practical and efficient technique for solving nonlinear PDEs such as the KdV equation. Its simplicity, coupled with its effectiveness, makes it a valuable tool in both theoretical and applied mathematics, particularly in problems where exact solutions are rare or difficult to obtain.

5.3. Future work

Future work in applying the Adomian Decomposition Method to the fractional Korteweg-de Vries (kdv) equation has the potential to significantly enhance both theoretical and understanding and practical application in varies fields of science and engineering . By pursuing those avenues researchers can contribute to deeper understanding of nonlinear wave phenomena and improve the tools available for analyzing complex systems.

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