WALLAGA UNIVERSITY
SCHOOL OF GRAUATE STUDIES

THE POWER SPECTRUM OF TWO-LEVEL LASERS OPERATING BELOW THE THRESHOLD

MSC. THESIS PROPOSAL

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Approval Sheet for Submitting Thesis Proposal

Title:-THE POWER SPECTRUM OF TWO-LEVEL LASERS OPERATING BELOW THE THRESHOLD

Submitted by

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Name of Student	Signature	Date

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Chapter 1

INTRODUCTION

1.1 Background of the study

The quantum interaction of light and matter is a fundamental area of research in physics that spans across its various fields, encompassing: quantum optics (Qin et al., 2024). Accurate measurement of the noise of semiconductor lasers establishes a reliable benchmark for studying and analyzing low-noise semiconductor lasers. Understanding the spectral characteristics of semiconductor laser noise is essential for developing appropriate measurement methods for different types of noise, because noise generation mechanisms vary across different frequency ranges (Wang et al., 2024). A two-level laser is a source of coherent or chaotic light emitted by two-level atoms inside a cavity coupled to a vacuum reservoir via a single-port mirror. In one model of such a laser, two-level atoms initially in the upper level are injected at a constant rate into a cavity and removed after they have decayed due to spontaneous emission. The two-level atoms available in a cavity are pumped to the upper level by some convenient means, such as electron bombardment. There has been considerable interest in studying the quantum properties of the light generated by a two-level laser. It is found that the light generated by this laser operating well above the threshold is coherent and the light generated by the same laser operating below the threshold is chaotic. In the quantum theory of a laser, the interaction of the atoms inside the cavity with the vacuum reservoir outside the cavity is usually taken into consideration (Kassahun, 2012).

The power spectrum of a two-level laser operating below threshold is characterized by a Lorentzian line shape centered at the atomic resonance frequency. The line width of the Lorentzian is determined by the spontaneous emission rate of the excited state. The power spectrum is proportional to the population difference between the excited and ground states, (Frauenheim et al., 2002) which is negative below threshold. The power spectrum can be calculated using the quantum Langevin equations, which describe the evolution of the laser field and atomic populations. The power spectrum can also be

measured experimentally using a spectrum analyzer(Allen and Eberly, 2012).
The power spectrum of a signal represents the distribution of its power over different frequencies. In the context of a two-level laser operating below threshold, the power spectrum would show a peak at the laser's resonant frequency, with the width of the peak determined by the laser's line width (Cavalieri et al., 1987).
The primary concept is that below threshold, a two-level laser emits spontaneous radiation with a power spectrum that is broad and smooth. This behavior is significant because it differentiates the operational characteristics of a laser system in different regimes, which impacts applications such as noise analysis and signal processing (Fischer et al., 2017).
The lasing threshold is the lowest excitation level at which a laser's output is dominated by stimulated emission rather than by spontaneous emission. Below the threshold, the laser's output power rises slowly with increasing excitation.

1.2 Statement of the problem

The power spectrum of a two-level laser operating below threshold typically shows a broadened spectral line due to insufficient gain and the influence of noise sources. As stated by (Cortés, West, and Lindenberg, 1985) the exact shape and width depend on the specific parameters of the laser system and the operating conditions. Although the relation between the frequency noise spectrum and the laser line shape has been studied extensively, no simple expression exists to evaluate the laser line width for frequency noise spectra that does not follow a power law. Therefore, this research will present a simple approach to this relation with an approximate formula for evaluation of the laser line width that can be applied to arbitrary noise spectral densities computational method

1.3 General objective of the study

. The general objective of this proposal is to study the characteristics of the power spectrum of two-level lasers operating below the threshold.

1.3.1 Specific objectives of the study

The specific objectives of this thesis proposal are to

- Study the behavior of a laser system that is not operating at its maximum output level

- Analyze the dependence of the power spectrum on pump rate, cavity losses and atomic linewidth.

- Study the factors that can influence spontaneous emission laser operations below threshold

- Develop algorithm for two-level lasers operating at the threshold using a power spectrum

Chapter 2

LITERATURE REVIEW

2.1 Two-level laser system and rate of equations

"LASER" is an acronym for "Light Amplification by Stimulated Emission of Radiation." The laser is a light source that exhibits unique properties such as directionality, mono chromaticity and coherence. These properties easily distinguish the laser from any other source of light hence has found a wide variety of applications in modern technology (Foth, 2008). Lasers have become a subject of research to date. In this essay the writer is going to discuss rate equations in a simple two - level laser. The transition rates of the three processes are: Spontaneous emission, Stimulated emission and Absorption. Lasers where the atomic system has only two energy levels involved in the lasing process, the minimum pump power required for the laser to start emitting coherent light. The distribution of power over different frequencies or wavelengths in the emitted light, below the threshold, the laser does not produce coherent light. Instead, it emits spontaneous emission, which is a random process. The power spectrum in this regime is dominated by spontaneous emission noise. Assume the laser parameters such as population inversion ΔN, cavity loss rate γ, and spontaneous emission rate Γ. The rate equations for a two-level laser below threshold are as follows

$$\frac{dN}{dt} = \Gamma(N_0 - N) - \gamma N \tag{2.1}$$

$$\frac{d\phi}{dt} = \Delta\omega \tag{2.2}$$

where N is the population inversion, ϕ is the phase, and $\Delta\omega$ is the detuning frequencyNeumeyer et al., 2019.

2.2 Noise sources in lasers

The noise arises from beating between the lasing mode and spontaneous emission into the sub-threshold modes on either side. The noise profiles are compared with gain profiles obtained for above-threshold laser amplification of an injected signal. Both sets of profiles broaden and shift with increasing laser power and appear identical in the case of symmetric sub-threshold modes. The spectra arise from beating between the lasing mode and spontaneous emission into two below-threshold modes. In addition to laser noise measurements, the relationship of noise with laser gain also (Hertzog et al., 2019) investigated. The laser is double ended to allow injection of a weak detuned signal at one end, and detection of the amplified signal which emerges from the other.

2.2.1 Spontaneous emission

Spontaneous emission is the process where an electron in an excited state decays to a lower energy state by emitting a photon without any external stimulation. Laser light sources now give the possibility of easily saturating atomic transitions and these saturation effects must be described correctly to obtain a quantitative understanding of the characteristics of the fluorescence light (like spectral distribution). When the spectral width of the light wave is very large, which is the case in thermal light sources and some multimode lasers (operating in a great number of closely spaced free-running modes), a perturbative treatment of the interaction between atoms and photons is possible. (Avan and Cohen-Tannoudji, 1977).
Two-level emitters are the main building blocks of photonic quantum technologies and are model systems for the exploration of quantum optics in the solid state. Most interesting is the strict resonant excitation of such emitters to control their occupation coherently and to generate close to ideal quantum light, which is of utmost importance for applications in photonic quantum technology. To date, approaches and experiments in this field have been performed exclusively using bulky lasers, which hinders the application of resonantly driven two-level emitters in compact photonic quantum systems (White, 2009).
As stated by (Lippi, 2021) Amplified Spontaneous Emission (ASE) is a pervasive phenomenon ever present in any optical system that exhibits gain. From the laser point of view, fundamental aspects of ASE begin with the realization of the existence of a threshold for the spontaneous amplification (Casperson, 1977).

2.2.2 Thermal noise

Thermal noise of optical reference cavities sets a fundamental limit to the frequency instability of ultrastable lasers. As stated by (Kessler, Legero, and Sterr, 2011) ultrastable optical cavities have become a standard tool for stabilizing laser systems needed

for high-resolution spectroscopy, optical clocks, optical microwave generation, and coherent optical frequency transfer. Thermal noise in laser mirrors has been investigated (Levin, 1998) and (Ni, 2024) in great detail with respect to gravitational wave detection. The standard expressions for the noise that is due to photon fluctuations in thermal background radiation typically apply only for a single detector and are often (Zmuidzinas, 2003) strictly valid only for single-mode illumination. The use of scattering matrices allows us to describe thermal noise in linear electrical networks equally well.
In quantum theory, the classical complex wave amplitudes are replaced by operators.

2.3 Power spectrum density analysis techniques

The power spectrum describes the distribution of power across different frequencies in a signal. For a laser below threshold, the spectrum shows spontaneous emission noise. The dynamics of a two-level laser involves the interaction of two-level atoms with a resonant cavity mode, the decay of the two-level atoms due to spontaneous emission, and the damping of the cavity mode by a vacuum reservoir.Thus taking into account these three components of two-level laser dynamics, we first obtain the equation of evolution of the laser density operator from which the Fokker-Planck equation for the P function is derived. The resulting equation is then essentially used to calculate the photon statistics, the power spectrum, and the spectrum of intensity fluctuations of the light generated by a two-level laser (Khanin, 2012).
The power spectrum $S(\omega)$ is given by:

$$S(\omega) = \frac{A}{(\omega - \omega_0)^2 + (\Gamma/2)^2} \tag{2.3}$$

where ω_0 is the central frequency, Γ is the linewidth, and A is a proportionality constantKassahun, 2011.

2.4 Analysis of existing models and their limitations

Dynamic interference (DI) of photoelectrons, a simple double-slit interference at the atomic scale, is of fundamental importance in the physics of atoms and molecules interacting with short and intense light pulses (Tóth, Borbély, and Csehi, 2023). A clear manifestation of DI in the photoelectron spectrum is the transformation of the single photo peak into a multi peak pattern upon increasing light-matter coupling. This is attributed to the interference of electron amplitudes generated with the same energy but with a certain time delay, at the rising and falling edges of the laser pulse. For ground state atoms, dynamic interference occurs in the strongly non-perturbative region, and turns out to be closely intertwined with the onset of atomic stabilization (Jiang and Burgdörfer, 2018).

Chapter 3

MATERIALS AND METHOD

3.1 The theoretical model

Mathematical Tools Software Matlab (Matlab 2018a Software) is a higher programming language that will be used in this study. MATLAB can acquire, analyze, and visualize data, and then export the results. The mathematical tools we intended to solve and simulate our correlation of material. To analyze the power spectrum, the Fourier transform to the resulting output over time using MATLAB's FFT function will be applied. After setting up and executing the provided MATLAB code, we will obtain plots showing the populations of the laser system and its power spectrum, which provides insights when the laser operates below the threshold. The theoretical model will likely involve solving the laser rate equations, incorporating langevin noise terms to account for fluctuations. The power spectrum will be obtained by calculating the Fourier transform of the intensity auto correction function. After identifying key features such as linewidth, peak power, and spectral shape, the results will be correlated with the theoretical predictions for two-level lasers below threshold.

3.2 Writing documents

Window 8 and LaTeX, is used in writing this thesis proposal, thus for final writing thesis we will use it since it fits standardized to write documents like books, or an article that is to submit for publication in a research journal. We will use it properly, it produces beautiful documents and maintain its originality so as we use Tex documents that can be printed on paper, but recently it is also widely used to produce PDF files that can be viewed on computer screens.

Chapter 4

EXPECTED OUTPUT

The laser is operating where the gain is insufficient to overcome losses, resulting in a lower output power. A model which can convert the time-domain rate equations to frequency-domain to obtain the power spectrum will be identified. A Mathematical model that Use the rate equations and boundary conditions to derive the power spectrum will be identified.

Chapter 5

WORK PLAN

The study activities are Shown in the table below: It cover time based activities from September 202024-June 2025

Table 5.1: Time table for the research

NO	Description	Sep-Dec, 2024	Jan-June, 2025
1	Selection and Title approval	X	-
2	Submitting written proposal	X	-
3	Proposal and Defend	X	-
4	Analysis and Writing final Thesis	-	X
5	Plagiarism test and correcting a comment	-	X
6	Defense of the thesis and Final Submission	-	X

Chapter 6

BUDGET BREAKDOWN

Table 6.1: All over budget breakdown

No	*Description*	*unit*	*UnitPrice*	Total Price
1	Paper	pkt	800	800
2	Binding research and proposal	2	1500	3000
3	Research payments	by term	8000	8000
4	Transportation Fees	Trip 2	1000	2000
5	Software Installation and data analysis	agreement	5000	5000
6	Perdiem for Students	round 1	6200	6200
7	Total Budget	–	22500	25000

Budget source:-Self

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