



**WALLAGA UNIVERSITY  
SCHOOL OF GRADUATE STUDIES**

**COLLEGE OF NATURAL AND COMPUTATIONAL SCIENCES  
DEPARTMENT OF PHYSICS**

**STUDY ON COUPLED AN OPTOMECHANICAL SYSTEMS FOR  
SENSING APPLICATIONS: DISPLACEMENT AND MASS**

**M.Sc Thesis**

**By: Nigatu Gurmessa Brusso**

**Advisor: Misganu Chewaka (Ph.D)**

**June 2023**

**Nekemte, Ethiopia**



**WALLAGA UNIVERSITY  
SCHOOL OF GRADUATE STUDIES**

**COLLEGE OF NATURAL AND COMPUTATIONAL SCIENCES  
DEPARTMENT OF PHYSICS**

**STUDY ON COUPLED AN OPTOMECHANICAL SYSTEMS FOR  
SENSING APPLICATIONS: DISPLACEMENT AND MASS**

**A Thesis Submitted to the College of Natural and Computational  
Sciences in Partial Fulfillment of the Requirements for the Degree of  
Master of Science in Physics (Quantum Optics).**

**By: Nigatu Gurmessa Brusso**

**Advisor: Misganu Chewaka (Ph.D)**

**June 2023  
Nekemte, Ethiopia**

# DECLARATION

This is to certify that this thesis entitled “Study on Coupled an Optomechanical Systems for Sensing Applications: Displacement and Mass” accepted in partial fulfillment of the requirements for the award of the degree in physics by the school of graduate studies, Wallaga University through the College of Natural Sciences done by Nigatu Gurmessa Brusso is a genuine work carried out by him under our guidance. The matter embodied in this thesis has not been submitted earlier for the award of any degree or diploma. The assistance and help received during the course of this investigation have been duly acknowledged. Therefore, I recommended that it can be accepted as fulfilling the thesis requirements.

**Misganu Chewaka (Ph.D)**

Major Advisor

-----

Signature

-----

Date



**WALLAGA UNIVERSITY**  
**SCHOOL OF GRADUATE STUDIES**  
**P.O. Box: 395, Nekemte, Ethiopia.**

**APPROVAL SHEET FOR SUBMITTING FINAL THESIS**

As members of the Board of Examining of the Final MSc. thesis open defense, we certify that we have read and evaluated the thesis prepared by Nigatu Gurmessa Brusso under the title ” **Study on Coupled an Optomechanical Systems for Sensing Applications: Displacement and Mass** ” and recommend that the thesis be accepted as fulfilling the thesis requirement for the Degree of Master of Physics in Quantum optics.

----- Chairperson	----- Signature	----- Date
----------------------	--------------------	---------------

----- Internal Examiner	----- Signature	----- Date
----------------------------	--------------------	---------------

----- External Examiner	----- Signature	----- Date
----------------------------	--------------------	---------------

**Final Approval and Acceptance**  
**Thesis Approved by**

----- Department PGC	----- Signature	----- Date
-------------------------	--------------------	---------------

----- Dean of College	----- Signature	----- Date
--------------------------	--------------------	---------------

**Certification of the Final Thesis**

I hereby certify that all the correction and recommendation suggested by the board of examiners are incorporated into the final thesis entitled “ Study on Coupled an Optomechanical Systems for Sensing Applications: Displacement and Mass ” by Nigatu Gurmessa Brusso

----- Dean of SGS	----- Signature	----- Date
----------------------	--------------------	---------------

## **DEDICATION**

I dedicate this thesis manuscript to my friends, brother Daniel Hundero and families for nursing me with affection and love and for their dedicated partnership in the success of my life.

## **BIOGRAPHICAL SKETCH**

The author was born on September 03, 1987 E.C in Metekel zone of Benshingual Gumze regional state specially Bullen worda. He attended his primary school at Amanji primary school and secondary school at Bullen General higher secondary school. After successfully passing Ethiopia school living certificate examination. He joined Wallaga University, department of Physics in 2006 E.C and graduated with bachelor of applied in physics in 2010 E.C.

Up on completion he been employed as a physics teacher in Metekel zone, Bullen secondary schools. After 4 years of service, he joined Wallaga University, school of graduate studies, and Department of Physics for perusing his Msc. by Quantum Optics in 2014 E.C.

## STATEMENT OF THE AUTHOR

I Mr. Nigatu Gurmessa Brusso hereby declare and affirm that the thesis entitled “Study on Coupled an Optomechanical Systems for Sensing Applications: Displacement and Mass” is my own work conducted under the supervision of Misganu Chewaka (Ph.D). I have followed all the ethical principles of scholarship in the preparation, data collection, data analysis and completion of this thesis. All scholarly matter that is included in the thesis has been given recognition through citation. I have adequately cited and referenced all the original sources. I also declare that I have adhered to all principles of academic honesty and integrity and I have not misrepresented, fabricated, or falsified any idea / data / fact / source in my submission. This thesis is submitted in partial fulfillment of the requirement for a degree from the Post Graduate Studies at Wallaga University. I further declare that this thesis has not been submitted to any other institution anywhere for the award of any academic degree, diploma or certificate.

I understand that any violation of the above will be cause for disciplinary action by the University and can also evoke penal action from the sources which have thus not been properly cited or from whom proper permission has not been taken when needed.

**Name: Nigatu Gurmessa Brusso**

Signature \_\_\_\_\_

Date: \_\_\_\_\_

Department: Physics

College: Natural and Computational Sciences

## **ACKNOWLEDGMENT**

First and foremost, I give special thanks and glory to the almighty God for giving me the strength, wisdom and good health to complete this thesis.

Secondly, my deepest and heartfelt gratitude goes to my advisor Misganu Chewaka (Ph.D) for his invaluable advice, guidance, constructive comment, encouragement, continuous support and friendly approach throughout this study. He serves as my source of inspiration as I work toward a profession in quantum optics. For the remainder of his life, I sincerely wish him success, tranquility, and elegance.

Thirdly, I would like to thank Wallaga University for creating favorable conditions for my studies. In particular, I want to thank the Physics department's faculty for their wonderful, peaceful teaching and learning methods and for creating favorable circumstances for my master's program in quantum optics. Last but not least, I want to express my genuine gratitude to my family and friends for their spiritual support, financial assistance and sincere encouragement.

# Contents

<b>DECLARATION</b>	<b>i</b>
<b>DEDICATION</b>	<b>iii</b>
<b>STATEMENT OF THE AUTHOR</b>	<b>v</b>
<b>ACKNOWLEDGMENT</b>	<b>vi</b>
<b>List of Figures</b>	<b>ix</b>
<b>List of tables</b>	<b>ix</b>
<b>ABBREVIATIONS</b>	<b>x</b>
<b>ABSTRACT</b>	<b>xi</b>
<b>CHAPTER ONE</b>	<b>1</b>
<b>1 INTRODUCTION</b>	<b>1</b>
1.1 Background of the study . . . . .	1
1.2 Statement of the problem . . . . .	7
1.3 Research questions . . . . .	7
1.4 Objective of the Study . . . . .	8
1.4.1 General Objective . . . . .	8
1.4.2 Specific Objectives . . . . .	8
1.5 Significant of the study . . . . .	9
1.6 Organization of Thesis . . . . .	9
<b>CHAPTER TWO</b>	<b>11</b>
<b>2 REVIEW LITERATURE</b>	<b>11</b>
2.1 Quantum sensing . . . . .	13
2.2 Optomechanical sensing applications . . . . .	14
2.2.1 Displacement sensing . . . . .	14
2.2.2 Mass sensing . . . . .	15
2.2.3 Atomic force microscopy . . . . .	18
2.3 Basic tools and techniques of optomechanics . . . . .	21
2.3.1 Hamiltonian formulation of harmonic oscillator . . . . .	24
2.3.2 Quantum langevin equations . . . . .	26
<b>CHAPTER THREE</b>	<b>29</b>

<b>3</b>	<b>MODEL AND METHODOLOGY</b>	<b>29</b>
3.1	Source of data . . . . .	30
3.2	Methods of data analysis and interpretation . . . . .	30
	<b>CHAPTER FOUR</b>	<b>31</b>
<b>4</b>	<b>RESULTS AND DISCUSSIONS</b>	<b>31</b>
4.1	Cavity optomechanical coupling . . . . .	31
4.2	Physical principles of cavity optomechanical sensing . . . . .	34
4.2.1	Optomechanical coupling . . . . .	34
4.3	Cavity of optomechanical systems . . . . .	35
4.4	Optomechanical sensing applications . . . . .	42
4.4.1	Noise, sensitivity, and bandwidth . . . . .	42
4.4.2	Displacement sensitivity . . . . .	45
4.4.3	Mass sensitivity . . . . .	47
	<b>CHAPTER FIVE</b>	<b>52</b>
<b>5</b>	<b>CONCLUSIONS AND RECOMMENDATIONS</b>	<b>52</b>
5.1	Conclusions . . . . .	52
5.2	Recommendations for future work . . . . .	53
	<b>References</b>	<b>54</b>
	<b>APPENDIX</b>	<b>59</b>

# List of Figures

Figure 1.1 – Schematic of general optomechanical system . . . . .	5
Figure 2.1 – Optomechanical mass sensing systems . . . . .	17
Figure 2.2 – AFM image scanning . . . . .	20
Figure 4.1 – Schematic of general optomechanical system . . . . .	35
Figure 4.2 – A laser-driven cavity . . . . .	37
Figure 4.3 – Zero point fluctuation ( $X_{ZPF}$ ) versus the optical (mechanical) resonance frequency ( $\omega_m$ ) . . . . .	40
Figure 4.4 – Optical-cavity-based displacement sensor . . . . .	46
Figure 4.5 – Resonance frequency versus effective mass ( $m_{eff}$ ) . . . . .	49
Figure 4.6 – Schematic of the mass sensor made of an array of mechanically coupled microcantilever beams . . . . .	51

## ABBREVIATIONS

NEMS	:	Nano electromechanical systems
SQL	:	Standard quality limit
COMS	:	Cavity optomechanical system
AFM	:	Atomic Force Microscope
MRFM	:	Magnetic Resonance Force Microscope
WGM	:	Whispering Gallery Mode
MEMS	:	Micro Electromechanical Systems
WPP	:	Wedge prism pair
PID	:	Photo ionization detector
CM	:	Cavity Mirror
PZT	:	Piezoelectric scanner
F-P	:	Fabry-Perot

## ABSTRACT

This thesis was aimed at studied the cavity coupled optomechanical systems for sensing applications; we study the progress of precision sensing applications using optomechanical systems. The method used in this study was first we would introduce the physical principles of optomechanical sensing, including a discussion of the noises and sensitivity of the systems and Hamiltonian of cavity optomechanics then we would examine the advancements in displacement sensing , mass sensing and introduce the progress of using quantum techniques especially laser light to enhance the performance of the optomechanical sensors. The result of the study was calculated using cavity of Hamiltonian, zero point fluctuation of mechanical oscillator and described a mechanical resonator by a harmonic oscillator, which has an effective mass  $m_{eff}$ , a spring constant  $k$ , and a resonance frequency. Additionally, wolfram mathematical soft wares has been used for drawing the graphs of resonance frequency versus effective mass and Zero point fluctuation versus the optical (mechanical) resonance frequency. Changes in the spring effective mass  $m_{eff}$  will cause the resonance frequency shift of the mechanical resonator  $f_m$  i .e as the resonance frequency increase the spring effective mass of the mechanical resonators decrease and as zero point fluctuation increase the optical (mechanical) resonance frequency decrease. The researcher of this study recommended experimental research for further investigations there by the concerned body provide necessary facility and equipment's.

**Keywords:** Cavity optomechanics, Mechanical resonators, Radiation pressure, Precision sensing, Laser light.

# CHAPTER ONE

## 1 INTRODUCTION

### 1.1 Background of the study

The field of optomechanics studies the interaction between optical and mechanical degrees of freedom mediated by radiation pressure force. The use of optical microcavities can greatly enhance this optomechanical interaction (Zhu et al., 2020).

Optomechanical cavities, which considerably enhance radiation pressure coupling between the optical and mechanical domains, are the core component of optomechanical systems. The optical resonance frequency is hence incredibly sensitive to mechanical motion. Also, the optomechanical cavities allow for a previously unheard-of decrease in the sensors' footprint (Dong, Fiore, Kuzyk, & Wang, 2012).

Optomechanics have attained ultrasensitive performance in gravitational wave detection, as well as high-precision measurements detection for mass, acceleration, displacement, and force (Clerk & Marquardt, 2014). Optomechanics turns out to be a useful for exploring fundamental physics and for using its many ideas in a variety of contexts, particularly in the context of so-called quantum technologies (Ding et al., 2010). Quantum mechanics can be put to the test in an entirely new area using optomechanics. Understanding the so-called transition barrier between the classical and quantum worlds, or vice versa, is one of the exciting research issues where optomechanics may be helpful. Since, the field of optomechanical applications, particularly

in the area of precise sensing, has rapidly evolved. While the current developments in optomechanical system-based precision sensing applications, such as displacement sensing, mass sensing, and atomic force microscope (AFM), would be the main focus of this review (Dobrindt, Wilson-Rae, & Kippenberg, 2008).

Cavity optomechanics is an evolving field aiming to explore and exploit the nonlinear radiation-pressure coupling between the cavity mode and mechanical degrees of independence (Fiore et al., 2011) to quantum information processing (Stannigel et al., 2012). Cavity optomechanical systems are displaying a variety of sensing and transduction applications, benefiting from improvements in related theories and experiments, which also encourages research based on quantum mechanics that is based on them (Metcalf, 2014).

The study of the quantum mechanical features of macroscopic mechanical resonators employed radiation pressure strength of photons to regulate and alter motions of mechanical resonators, with a focus on the underlying physics and quantum effects (Chan et al., 2011).

Because to their improved mechanical and optical resonance responses and readout sensitivity, COMS also offer the best stages for precision sensing. Using cavity optomechanical systems, it has been possible to understand shot noise limited displacement sensitivity on the order of  $10^{-19} m/\sqrt{Hz}$ . Moreover, COM sensors benefit from being compact, light, and power-efficient. They can also be incorporated on a chip and work with fiber coupling, among other things. As a result, COM sensors have a big chance to be used in practical applications in the near future because to their benefits

of small size, low mass, and influence consumption, on-chip integration capabilities, compatibility with fiber coupling, etc. (Krause, Winger, Blasius, Lin, & Painter, 2012).

Cavity optomechanics can be used to process quantum information, or to store and send information. To process the information, one can attach a superconducting transistor to a mechanical system and subsequently an optical system. Sensitive force (more specifically, force gradients) detection. Because if we apply a force gradient to an oscillator, it will add some effect to the oscillator's spring constant and modify the frequency, which is obvious. A cantilever's weight can change slightly with a given frequency, allowing for ultrasensitive mass detection that is helpful for chemical diagnostics. We can arrange some molecules on the cantilever's surface so that it only accepts or binds with other particular moles. As a result, if there is a flow of gas molecules, only molecules of a given species will stick to the cantilever and change its mass. Using this concept, different chemical species can be found. displacement detection with extreme sensitivity (using quantum sensors). The best part is that these gadgets can be made using nanotechnology and can have everything integrated on a single chip (Guzmán Cervantes, Kumanchik, Pratt, & Taylor, 2014).

We would focus on the most recent developments in optomechanical system-based precision sensing applications, such as mass and displacement sensing.

The Fabry-Pérot resonator is made up of two extremely reflective mirrors with an L-shaped space between them. one end mirror of them is immobile, while the other is flexible and resembles a mechanical oscillator. Number of

resonances with eigen frequencies are supported by this cavity  $\omega_{c,n} \approx n\omega_c$  with  $\omega_c = \frac{\pi c}{L}$  here  $n$  is the figure mode number and  $c$  is the speed of light in vacuum. For brevity, we reflect a single cavity mode with frequency  $\omega_c$  (Wollman et al., 2015).

The input laser typically enters the cavity, passes through the fixed mirror, and then is reflected by the moving mirror. Its reflection changes the moveable mirror's location and enables momentum to be transferred from photons to mechanical modes, which exerts restriction pressure on the mechanical system's degree of freedom. The optical resonance frequency is also changed by the mechanical motion, which likewise affects how much light is circulated and how well it radiates. Optomechanical coupling is achieved in this way by applying radiation pressure to create a dynamic coupling between the mechanical oscillator and optical field (Aspelmeyer, Kippenberg, & Marquardt, 2014).

We run a complete quantum description of cavity optomechanical interaction below to precisely characterize the coupling process and the complete Hamiltonian:

$$\hat{H} = \hat{H}_0 + \hat{H}_{int} + \hat{H}_{dri} \quad (1.1)$$

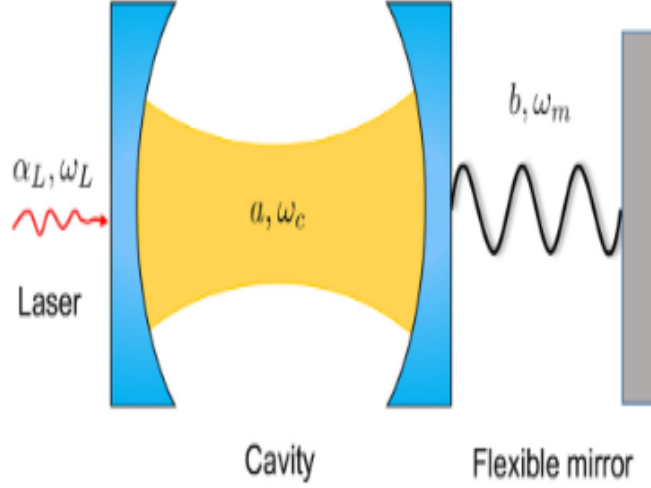


Figure 1.1: Schematic of general optomechanical system  
(Zhu et al., 2020)

Schematic of an optomechanical Fabry-Perot resonator with a mirror that can be shifted from one end to the other (right) (left). After reflecting a photon, the moveable mirror recoils as a result of radiation pressure. Where  $\hat{a}(\hat{a}^+)$  and  $\hat{b}(\hat{b}^+)$  are the annihilation (creation) operators of the optical cavity mode and the mechanical mode, respectively. Where the free Hamiltonian for an uncoupled mechanical oscillator with a single cavity mode is as follows:

$$\hat{H}_{\text{free}} = \omega_c \hat{a}^\dagger \hat{a} + \omega_m \hat{b}^\dagger \hat{b} \quad (1.2)$$

We define the position and momentum operators as (Allain et al., 2020).

$$\hat{X}_o = x_{zpf} (\hat{b} + \hat{b}^\dagger) \quad (1.3)$$

$$\hat{P}_0 = -im\omega_c x_{zpf} (\hat{b} - \hat{b}^\dagger) \quad (1.4)$$

where  $m$  effective mass of mechanical oscillator,  $x_{zpf} = \sqrt{\frac{\hbar}{2m\omega_c}}$  is the zero-point fluctuation amplitude of the mechanical oscillator. The above position

and momentum operators satisfy the commutation relation

$$[x_{zpf}(\hat{b} + \hat{b}^\dagger), -im\omega_c x_{zpf}(\hat{b} + \hat{b}^\dagger)] = [\hat{X}_0, \hat{P}_0] = i\hbar \quad (1.5)$$

The right end mirror's movement changes the cavity length, causing the coupling of the optical and mechanical modes to be parametric, meaning that the mechanical amplitude modulates the cavity resonance frequency. In the Taylor series, increase the optical frequency as (Aldana, Bruder, & Nunnenkamp, 2013):

$$\omega_c(x) = \omega_c(0) + \frac{x(\partial\omega_c)}{\partial x} + o(x) \approx \omega_c(0) + g(\hat{b} + \hat{b}^\dagger) \quad (1.6)$$

where  $g_0 = \left[\frac{\partial\omega_c}{\partial x}\right] x_{zpf}$  is the radiation-pressure coupling strength (or single-photon optomechanical coupling strength). Thus the interaction part of the Hamiltonian can be written as:

$$\hat{H}_{\text{int}} = \hbar g_0 \hat{a}(\hat{a}^\dagger)(\hat{b} + \hat{b}^\dagger) \quad (1.7)$$

In addition, the last term in  $\hat{H} = \hat{H}_0 + \hat{H}_{\text{int}} + \hat{H}_{\text{dri}}$  describes the input optical driving of the system, with the Hamiltonian:

$$\hat{H}_{\text{dri}} = \alpha_L(\hat{a}e^{i\omega_L t} + \hat{a}^\dagger e^{-i\omega_L t}) \quad (1.8)$$

Here  $\omega_L$  is the frequency of input driving laser  $\alpha_L = \sqrt{\frac{K_{ex} p_{in}}{\hbar\omega_L}} e^{i\phi}$  represent the driving strength, where  $p_{in}$  is the input laser power,  $K_{ex}$  refers to the loss rate associated with input coupling, and is the initial phase of input laser. In a reference frame rotating at the laser frequency, the total Hamiltonian

$\hat{H} = \hat{H}_0 + \hat{H}_{int} + \hat{H}_{dri}$  of the optomechanical system can be written as:

$$\hat{H} = -\Delta\hat{a}(\hat{a}^+) + \omega_m\hat{b}(\hat{b}^+) + g_0\hat{a}(\hat{a}^+)(\hat{b} + \hat{b}^+) + \alpha_L(\hat{b} + \hat{b}^+) \quad (1.9)$$

Where  $\Delta = \omega_L - \omega_c$  is the detuning of the input laser and cavity frequency (Riedinger et al., 2018)

## 1.2 Statement of the problem

Cavity optomechanical systems play a role in the creation of cutting-edge applications and significant quantum mechanics research. Due to the macroscopic quantum characteristics of mechanical resonators, cavity optomechanical systems have a tremendous potential for studying basic problems in quantum theory (Kim et al., 2017). Cavity optomechanics was introduced as the most recent "innovative of photon history" in nature photonics along well documented concepts and technology like quantum information. The skill to achieve the cavity coupled optomechanical systems for sensing applications: Displacement and Mass is difficult to reach for great determination cases will and how optomechanical sensing applications such as Displacement sensing and Mass sensing work.

While this analysis would focus on the new examination progresses of optomechanical system based precision sensing applications, including displacement sensing, mass sensing.

## 1.3 Research questions

The following questions may arise for further study from the above mentioned phenomenon:

✓ What are the recent Physical principles of cavity optomechanical sensing?

✓ What is the cavity coupled optomechanical systems sensing applications: displacement and mass?

✓ What are the principles of cavity coupled optomechanical systems sensing application: displacement and mass?

## **1.4 Objective of the Study**

### **1.4.1 General Objective**

The general objective of this work was to study coupled an optomechanical systems for sensing applications.

### **1.4.2 Specific Objectives**

To attain the general objectives, the following specific objectives were employed:

\* Principles of coupled an optomechanical systems for sensing applications of Displacement was determine.

\* Principles of coupled an optomechanical systems for sensing applications of Mass was determine.

\* The progress of precision sensing applications: Displacement and Mass using coupled an optomechanical system was analysed

\* The noise sensitivity of cavity coupled optomechanical systems was identified

## **1.5 Significant of the study**

Currently dynamic efforts are under way to cavity coupled optomechanical sensing and the cavity coupled Optomechanical sensing applications. However, they do not lend to understanding of the Physical principles of cavity coupled optomechanical sensing and the cavity coupled optomechanical sensing applications.

This understanding is important in order to develop the principles of optomechanical coupling, optomechanical sensing platforms, and noise, sensitivity, and bandwidth and the application of cavity coupled optomechanical sensing of displacement sensing and cavity coupled optomechanical sensing of mass sensing. We were explore the different principles of Physical cavity coupled optomechanical sensing and the cavity coupled optomechanical sensing applications: Displacement and Mass. The outcome of the study also helps all scientific communities, industries and new coming investigators being the data used as reference for additional related investigations.

## **1.6 Organization of Thesis**

This thesis contains 5 chapters and is organized as: Chapter one provides introduction and necessary preliminaries concepts of the cavity coupled optomechanical systems for sensing applications: Displacement and Mass. Such as: principles of optomechanical coupling, noise analysis and sensing towards quantum limit, optomechanical sensing platforms, and optomechanical sensing applications which would be used throughout this study. Chapter two provides a review of related literature, Chapter three reveals the methodology and mathematical procedure used to conduct the thesis on

the given title, Chapter four discussion and Results. Finally, Chapter five presents the brief recommendation for future works and conclusion of the study.

# CHAPTER TWO

## 2 REVIEW LITERATURE

With the discovery of the connection between the optical field and mechanical oscillation, the field of optomechanics has quickly advanced in study the nonlinear optomechanical interaction was predicted to have extensive and nontrivial consequences on cavity optomechanical systems (Zhu et al., 2020).

The linearization of the optomechanical interaction, however, has received the majority of attention in recent years, ignoring the optomechanical coupling's intrinsic nonlinearity. In both classical and quantum systems, determining nonlinear optomechanical collaboration is becoming increasingly important, and nonlinear optomechanical collaboration has emerged as a significant new field in cavity optomechanics (Xu, Jiang, Clerk, & Harris, 2019).

It allows for a wide range of applications, including the creation of nonclassical states and single-photon sources provides a brief study of these developments and a discussion of a few recent difficulties in this area (O'Connell et al., 2010). In order to characterize phenomena that are beyond first order in the radiation pressure coupling, the two methods for obtaining corrections to the cavity optomechanics "linear model" are contrasted by (Bowen & Milburn, 2015).

A popular phenomenological Hamiltonian that conserves the photon number is compared to a two-mode truncation in the zone where the mechanical

frequency is significantly lower than the cavity frequency (Lecocq, Clark, Simmonds, Aumentado, & Teufel, 2015).

The Hamiltonian of Law's microscopic model, which we consider to be the "real" system, is contrasted. Although these methods concurred at first order, the latter model does not conserve the photon number, making computations difficult (Stannigel et al., 2012) .

For quantitative precision content analysis and high-yield mass production of many kinds of materials, mass sensing with a broad detection range and high resolving power is crucial (Dobrindt et al., 2008) .

Due to the radiation pressure interaction between its optical and mechanical modes, cavity optomechanics offers a strong platform for the observation of numerous intriguing classical and quantum nonlinear events (Guzmán Cervantes et al., 2014). Because of its significance in both fundamental physics and possible applications, including hidden information processing and optical communications, the chaos caused by optomechanical nonlinearity has received a lot of attention in particular.

The link between optical and mechanical modes is found by COM to be mediated by the radiation pressure force. In contrast to the passive method, active optomechanics with optical gain directly imposes mechanical motion upon the lasing dynamics, revealing the intrinsic features specified by the system itself (Clerk & Marquardt, 2014).

The quantum character of atom cavities and phonon-photon interactions is highlighted by lowering the optical gain to one active atom. the oneatom optomechanical micro laser produces non-classical photon-phonon pairs in

addition to non-classical photons. Cavity optomechanics to an active mode, paving the path for optomechanical light sources for photonic integrated circuits, onchip quantum communication, and bio sensing (Stannigel et al., 2012).

Cavity optomechanical systems are displaying a variety of applications in sensing and transduction, benefiting from advancements in related theories and experiments, which also supports quantum research based on them. Introduces the typical applications of cavity optomechanical systems, some recent advancements in this area, and then discusses the potential of cavity optomechanical systems for examining fundamental quantum theory questions as well as the difficulties faced in recent developments (Gavartin, Verlot, & Kippenberg, 2012) . Cavity optomechanical systems are essential to quantum computing, enhance the quantum toolkit, and are especially useful for quantum interfaces and quantum memory (Giessibl, 2003).

## **2.1 Quantum sensing**

One of the following is commonly used to explain quantum sensing: measuring a physical quantity with the aid of a quantum item (classical or quantum). Quantized energy levels define the quantum object, the electronic, magnetic, or vibrational states of superconducting or spin qubits, neutral atoms, or trapped ions are specific examples, the measurement of a physical quantity using quantum coherence or wavelike spatial or temporal superposition states and using quantum entanglement to increase a measurement's sensitivity or accuracy beyond what is feasible conventionally (Gefen, Rotem, & Retzker, 2019).

## 2.2 Optomechanical sensing applications

### 2.2.1 Displacement sensing

The foundation of many other sensing applications is high-sensitive displacement sensing since variations in numerous other physical parameters can result in component position change. Typically, the mechanical displacement can be read out by a photo detector or an optical spectrum analyzer by converting it to an optical transmission signal through the optomechanical coupling coefficient (Allain et al., 2020) .

The shot-noise-limited displacement sensitivity is improved by strengthening the optomechanical coupling. Shot noise is no longer the limiting constraint for the sensitivity of photonic crystal cavity systems thanks to the robust optomechanical connection. The optical transmission signal frequently contains information about the mechanical resonator's displacement thanks to optomechanical coupling. Then it can be read out by an optical spectrum analyzer and a photo detector (Xu, Mason, Jiang, & Harris, 2016).

Every application that requires the detection of movement, such as shifts in position, gaps and changes in gaps, uses displacement sensors. Vibrations, expansion caused by temperature changes, expansion of objects, and inclines. Moving objects are translated into electromagnetic, electrostatic, or magnetoelectric signals via displacement sensors, which are then read and transformed into data and presented to the user in a readable way. Displacement sensors are used in so many various industries, and there are a ton of applications in each of these businesses. Every movement will necessitate measurement, which necessitates the use of a displacement sensor of some sort (Forstner et al., 2012).

It has been established that the displacement sensor based on WGM cavity optomechanical systems has great sensitivity. By using polarization spectroscopy techniques to decrease the noise, a displacement sensor based on a cavity optomechanical system was able to attain the degree of displacement sensitivity that was previously only possible at the expense of shot noise (Schliesser, Anetsberger, Rivière, Arcizet, & Kippenberg, 2008).

Due to nano-opto-mechanical structures' high sensitivity, compact size, rapid working rate, and potential for integrated and hence parallel reading, recent breakthroughs in optomechanics have opened up interesting possibilities for displacement sensing but for scalable implementations of optical detectors and actuation must also be integrated (Metcalf, 2014).

### 2.2.2 Mass sensing

In the measurement of nanotechnology, mass sensors are crucial components for sensing nanoparticles, mass spectrometers are typically utilized (even single atoms). As of right now, mass detection uses nanomechanical oscillators. They already made it possible to measure mass with an incredibly fine level of sensitivity down to  $1 \times 10^{-21}$  g in vacuum and  $1 \times 10^{-15}$  g in liquid environment. The external examples with mass  $\Delta m$  depositing onto the surface of a mechanical resonator will lead to the resonant frequency shift  $\Delta\omega_m$  (Forstner et al., 2012),

$$\Delta\omega_m = \frac{-2m_{eff}}{\omega m} \quad (2.1)$$

$$f_m = \frac{1}{2\pi} \sqrt{\frac{K}{m_{eff}}} \quad (2.2)$$

Changes in the spring effective mass  $\delta m_{\text{eff}}$  will cause the resonance frequency shift of the mechanical resonator  $\delta f_m$ , which is  $\delta m_{\text{eff}} = \frac{2m_{\text{eff}}}{f_m} \delta f_m$ . Qualitatively, we simply consider the deposited mass

$$m_{d \approx \delta m_{\text{eff}}} = \frac{-2m_{\text{eff}}}{f_m} \delta f_m \quad (2.3)$$

The measurement of the mass of electrically neutral particles has developed into a popular research topic as a result of the necessity to detect molecular-scale substances in the fields (Lecocq et al., 2015). Microresonators are typically particularly sensitive to the disruption brought on by outside effects due to their tiny size. These mechanical resonator-based mass sensors do not require the molecular ionization process; instead, they rely on a resonant frequency shift brought on by an accreted mass. From Eq. (2.3), it is clear that smaller masses can be detected when the device's goal is immovable by lowering the mechanical resonator's real mass or increasing its resonance frequency. More precisely, the necessary position of the object in the mechanical mode, as described in (Sansa et al., 2020), also affects the relationship between statement mass and rate shift.

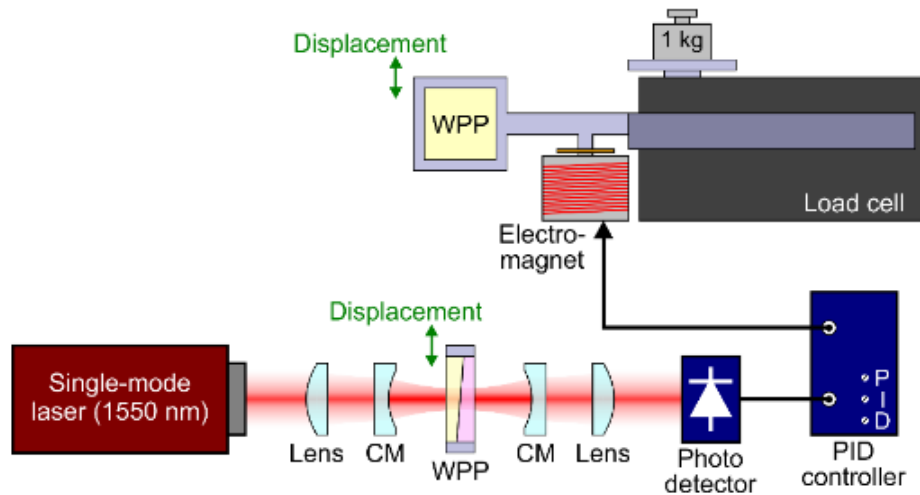


Figure 2.1: Optomechanical mass sensing systems (Alnis, Matveev, Kolachevsky, Udem, & Hänsch, 2008)

Schematic diagram of the optomechanical mass sensing system, where the opticalcavity-based displacement sensor and the load cell are combined together with an electromagnet and a PID controller to implement the negative feedback system and the null method-based mass measurement. A single mode laser beam at 1550 nm wavelength is sent to the free-space Fabry-Pérot optical cavity to detect the displacement of the WPP, which is allowed to change by the mass on the load cell and the voltage applied to the electromagnet. Note that the two WPPs displayed in the diagram actually correspond to a single device. CM, cavity mirror. We use the null method and a negative feedback system to further improve the accuracy of mass detection. Prior to applying a mass to the load cell, we established the WPP position's zero point by changing the magnetic voltage will center the cavity transmission (Alnis et al., 2008).

As soon as a mass is applied to the load cell, the WPP begins to move, and the ensuing change in cavity transmission results in the generation of a non-zero error signal. After receiving the error signal, a PID controller in the negative feedback system provides a voltage that is delivered to the

electromagnet, which generally causes the WPP to return to its initial zero point. The value of mass is determined by the magnitude of the electromagnet voltage at which the WPP is at rest at its original zero position (i.e., the error signal at the PID controller remains at zero). This null-method-based methodology, in particular, makes the mass measurement far less impacted by the non-uniformity of WPP thickness over the surface that can come from the aforementioned wedge angle mismatch or improper surface cleaning. demonstrates the general schematic diagram of our optomechanical mass sensing system (Rabenimanana et al., 2019).

The light source is a single-mode laser operating at a wavelength of 1550 nm, and the light is output to a confocal Fabry-Pérot optical cavity made of two concave mirrors with reflectivity and WPP. The laser light sent through the cavity is captured by a photo detector. The photo detector signal is supplied into a PID controller, which produces a voltage that is applied to the electromagnet for the mass measurement based on the null approach. To increase the stability of the entire system, we combine the load cell, the electromagnet, and all of the optical parts, including the cavity mirrors, onto a meticulously built monolithic base plate (Alnis et al., 2008).

### **2.2.3 Atomic force microscopy**

In comparison to the optical diffraction limit, the resolution of atomic force microscopy (AFM), a form of scanning probe microscopy (SPM), has been verified to be on the order of parts of a nanometer. By "sensing" or "touching" the surface with a mechanical examination, the data is acquired.

Piezoelectric components allow for small, exact movements under (electric) control, this allows for precision scanning. The Atomic Force Microscope does not actually use the nuclear force, despite its name (Chae et al., 2017)

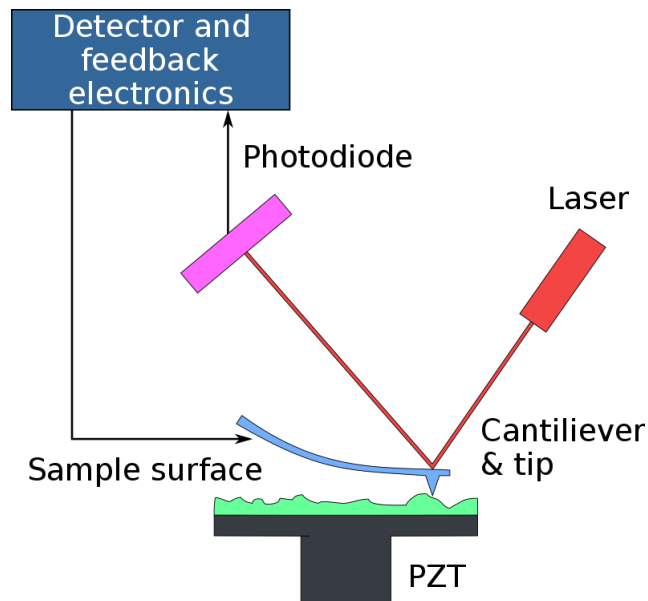


Figure 2.2: AFM image scanning  
(Chae et al., 2017)

By moving a tiny cantilever across the surface of a sample, an AFM creates pictures. The cantilever is bent when its pointed end makes contact with a surface, affecting how much laser light is reflected into the photodiode. The measured cantilever height then follows the surface when the height of the cantilever is changed to restore the response signal (Giessibl, 2003).

System for visual display that creates an image from the measured data. Due to the AFM's sensitivity, it is important to take precautions to guarantee that external vibrations, such as those from the laboratory building, do not impair its functionality. An undesired motion of the tip with regard to the sample and the deflection sensor is the result of external vibration. In theory, the force microscope ought to be capable of spotting individual electrons (Binnig, Gerber, Stoll, Albrecht, & Quate, 1987)

### 2.3 Basic tools and techniques of optomechanics

In stasics let us assume that the mechanical mirror is moved very, very slowly, akin to a static situation, i.e. for each position of the movable mirror, we give enough time to the mechanical oscillator to settle to the new state and in particular, for the light field intensity to adjust to the new length of the cavity. As the mirror moves along, at some point it will be in resonance with the incoming laser light and the cavity will be filled up with light. So the radiation pressure force will also increase(Eichenfield, Chan, Camacho, Vahala, & Painter, 2009).

When there is a force, the force pushes the mirror to the right, so we will have a dip and so on. This is a rather curious potential induced by the radiation pressure force. Please note that the full mechanical motion sees the standard restoring force potential plus this potential. One can see that at some places there is local minima. There are several such minima depending on the strength of the extra potential, provided by the light field intensity. The local minima of the overall potential as seen by the mechanical system correspond to stable equilibrium positions.

By changing some parameters, say the light intensity and looking at the output intensity, one can see that the system oscillates around a local minimum. However, it may also enter into a different local minimum and now if we reduce the light intensity it may stay in that minima rather than going back to the original minimum, giving rise to the phenomena of Hysteresis. This is happening owing to the existence of many equilibrium positions. Note that the original curvature of the potential (without the light field) gets changed owing to the presence of the light field. So the spring constant of

the mechanical oscillator now has an effective spring constant, as expressed in Equation (2.4). This is known as the optical spring effect (Bai, Wang, Zhang, Liu, & Wang, 2020).

$$V_t = V_{HO} + V_{RP} = \frac{1}{2}K_{eff}x^2 \quad (2.4)$$

In Dynamics Let us now relax the static case. The mechanical oscillator or the cantilever now moves with a finite speed. The light field does not get time to completely track the mechanical motion. So we will have time effects. The green line denotes the effect of time-lag. In the dynamic case, the force is no longer a function of position at a given instant of time as was the case with static:

$$F_{RP} \neq F_{RP}(x(t)). \quad (2.5)$$

But  $F_{RP}(t)$ , the force at a given time depends on the full pre-history of the motion of  $x$  because that determines how the light field builds up and depletes again.

$$F_{RP} = F_{RP}(x(t'), t' \leq t). \quad (2.6)$$

This force is no longer a conservative force. So we cannot write down a potential. It implies this force generates some friction. It may be noted here that even in thermal equilibrium, the harmonic oscillator undergoes some oscillations whose amplitudes change with time. Let us follow through one such cycle of these oscillations, say at position  $x_a$ . As we approach the resonance, the force is little bit smaller than earlier due to time lag (as it still

had to build up) and then as we go back, we have already reached the higher intensity and the force will remain high for some time than the adiabatic case. So the force in the two halves will not be identical in magnitude. So as a consequence, if we calculate the work done by the radiation pressure force in such a cycle, it will turn out to be non-zero. In fact,

$$\oint F_{RP} dx < 0. \quad (2.7)$$

This means, the force provided by the light field extracts energy from the mechanical motion, inducing extra damping in the mechanical oscillator. This is called light induced damping or optomechanical damping,  $\gamma_{om}$ . So, the total damping of the mechanical oscillator will be due to both the mechanical damping ( $\gamma$ ) and the optomechanical damping ( $\gamma_{om}$ ).

$$\gamma_t = \gamma + \gamma_{om} \quad (2.8)$$

Now, whenever there is extra damping but no extra fluctuations, then this damping can simply damp away the thermal fluctuations. Thermal fluctuations always arise as an equilibrium between damping that wants to extract energy and thermal random force that comes from outside that tries to heat up the system. In thermal equilibrium we have a balance between the two. So if we increase the damping we will be able to effectively reduce the thermal fluctuation (Dobrindt et al., 2008).

$$\begin{aligned} \oint F_{RP} dx < 0 &\Rightarrow \text{extra damping} \\ &\Rightarrow \text{light-induced cooling.} \end{aligned} \quad (2.9)$$

This is good news! Because the typical refrigerators in Lab are not good enough to cool mechanical systems down to the ground state. We can then exploit these extra mechanisms to cool the oscillator further down. Note that, the situation we have considered above has a cavity length smaller compared to the resonance one. This implies that:  $\omega_o > \omega_l$ , i.e. the laser is red-detuned. If we place the cantilever in the position,  $x_b$  (please refer to Figure (4)), we will have a situation where the cavity length is longer than the resonance one. It will correspond to the case,  $\Delta_0 > 0$ , i.e. the laser is blue detuned. In this configuration, the laser light will dump energy to the mechanical mirror, thereby heating it (Akram, Munro, Nemoto, & Milburn, 2012).

$$\begin{aligned} \Delta_0 < 0 : \text{Cooling}; \quad \gamma_{om} > 0 \\ \Delta_0 > 0 : \text{Heating}; \quad \gamma_{om} < 0 \end{aligned} \tag{2.10}$$

### 2.3.1 Hamiltonian formulation of harmonic oscillator

The total Hamiltonian,  $H$ , of a typical optomechanical system consists of the contributions from the optical components, the mechanical components, the interaction between the optics and the mechanics, the laser drive and the transfer of photons and phonons to and from the environment. Let us derive each of these terms one by one. The optical and mechanical modes can be represented as harmonic oscillators with annihilation operators  $\hat{a}$  and  $\hat{b}$  respectively. Each of these operators obey the Bosonic commutation relations  $[\hat{\mathcal{O}}, \hat{\mathcal{O}}^\dagger] = 1$  with their number operators given by  $\hat{n}_{\mathcal{O}} = \hat{\mathcal{O}}^\dagger \hat{\mathcal{O}}$ . Thus, one can write the energy of the optical (mechanical) mode in terms of the number of photons (phonons) times the energy of each quanta, and obtain the

combined Hamiltonian of the optical and mechanical modes as (Aspelmeyer et al., 2014).

$$\begin{aligned}\hat{H}_{om} &= \hbar\omega_c(\hat{x})\hat{a}^\dagger\hat{a} + \hbar\omega_m\hat{b}^\dagger\hat{b}, \\ &= \hbar\omega_o\hat{a}^\dagger\hat{a} + \hbar\omega_m\hat{b}^\dagger\hat{b} + \hat{H}_{int},\end{aligned}\tag{2.11}$$

where we have used Equation (4) to write the Hamiltonian describing the interaction between the optical and mechanical modes as

$$\hat{H}_{int} = -\hbar G\hat{x}\hat{a}^\dagger\hat{a} = -\hbar g_0(\hat{b}^\dagger + \hat{b})\hat{a}^\dagger\hat{a}.\tag{2.12}$$

Here, we have introduced the optomechanical coupling constant.

$$g_0 = Gx_{ZP} = G\sqrt{\frac{\hbar}{2m\omega_m}},\tag{2.13}$$

where  $x_{ZP}$  is the zero-point position and is typically of the order of  $10^{-15}$  m. This coefficient relates the radiation pressure force of a single photon on the position of a phonon and can range from a few Hz to a few MHz. We now add the laser drive term which describes a photon created inside the cavity at the laser frequency and its Hermitian conjugate. Thus, we obtain

$$\begin{aligned}\hat{H}_{sys} &= \hbar\omega_o\hat{a}^\dagger\hat{a} + \hbar\omega_m\hat{b}^\dagger\hat{b} - \hbar g_0\hat{a}^\dagger\hat{a}(\hat{b}^\dagger + \hat{b}) \\ &\quad + i\hbar A_l(\hat{a}^\dagger e^{-i\omega_l t} - \hat{a}e^{i\omega_l t}).\end{aligned}\tag{2.14}$$

The time-dependent terms in this Hamiltonian can be removed by switching to the rotating frame of the laser. This transformation also helps us to analyze the slower dynamics of the mechanical motion. We obtain the

Hamiltonian in this frame as

$$\begin{aligned} \hat{H}_{\text{sys}} = & -\hbar\Delta_0\hat{a}^\dagger\hat{a} + \hbar\omega_m\hat{b}^\dagger\hat{b} - \hbar g_0\hat{a}^\dagger\hat{a}(\hat{b}^\dagger + \hat{b}) \\ & + i\hbar A_l(\hat{a}^\dagger - \hat{a}), \end{aligned} \quad (2.15)$$

where  $\Delta_\omega = \omega_l - \omega_o$  is the laser detuning and the operator  $\hat{a}$  is written in the rotating frame. Typically, the resonant frequency of the cavity ( $\omega_o$ ) is on the order of  $10^{15}$  Hz, whereas the mechanical resonance frequency ( $\omega_m$ ) is usually in the range of MHz to GHz. The laser frequency is therefore adjusted in such a way that its detuning  $\Delta_0$  is comparable to  $\omega_m$ . Also, since the optical frequencies are very high, the effect of the thermal photons entering the cavity can be safely neglected and the cavity can be considered to be coupled with a reservoir at zero temperature. However, the laser drive does introduce noise into the system. On the other hand, thermal phonons play a very important role on the quantum dynamics of the mechanical mode. We explore these environmental effects systematically in the next section. Also, note that we shall drop the operator symbol in the rest of the article to make the expressions simpler.

### 2.3.2 Quantum langevin equations

Optomechanical cavities are open quantum systems. Such systems interact with the environment and undergo fluctuations and dissipation's , resulting in the decay of photons, mechanical damping as well as insertion of environmental noises(Bowen & Milburn, 2015). In dissipation Processes since the photons are coupled to the external environment through the mirrors, they experience absorption and scattering losses. These losses collectively define the optical decay rate  $\kappa$ . The optical decay rate also determines the qual-

ity factor of the cavity, which is given by  $Q_0 = \omega_o/\kappa$ . This factor signifies the total number of times a single photon oscillates inside the cavity before moving out. Similarly, the mechanical motion is affected by environmental factors such as viscous drag, clamping losses, phonon-phonon interactions and losses due to its composition. The mechanical damping rate  $\gamma$  takes these effects into account. It also characterizes the strength of coupling between the mechanical mode and the environment. The mechanical quality factor  $Q_m = \omega_m/\gamma$  is equivalently related to the phonon lifetime.

Input Fluctuations the environment also introduces noises into the system and steers the system dynamics. However, such a thermal environment could be modelled by a bath of several non-interacting harmonic oscillators. Effectively, the noises entering the system can also be approximated as Markovian, zero-mean and  $\delta$ -correlated. It is useful to note that a system is termed as Markovian if its present state does not depend on previous history. Moreover, zero-mean and  $\delta$ -correlated noise are known as white noise. Therefore, the white noise entering the cavity through the laser drive ( $a_{in}$ ) and the Langevin noise introduced into the mechanical oscillations ( $b_{in}$ ) follow the standard correlation relations(Gardiner & Collett, 1985)

$$\langle a_{in}(t)a_{in}^\dagger(t') \rangle = \delta(t-t') \quad (2.16)$$

$$\langle b_{in}^\dagger(t)b_{in}(t') \rangle = n_{th}\delta(t-t') \quad (2.17)$$

$$\langle b_{in}(t)b_{in}^\dagger(t') \rangle = (n_{th} + 1)\delta(t-t') \quad (2.18)$$

where  $n_{th} = [\exp\{\hbar\omega_m/(k_B T)\} - 1]^{-1}$  is the mean thermal occupancy at bath frequency  $\omega_m$  and temperature  $T$ .  $k_B$  is the Boltzmann constant.

Input-Output Relations Although the photon energy decay rate  $\kappa$  is a collective term, it is predominantly associated with the input-output losses than the intrinsic cavity losses. As such, the optical field amplitude effectively decays at the rate  $\kappa/2$ . Also, it is important to note here that input-output theory relates the laser power  $P_l$  entering the cavity to the amplitude inside it as  $A_l = \sqrt{\kappa P_l / \hbar \omega_l}$ . The input power associated with the white noise is given by  $P_{in} = \hbar \omega_l \langle a_{in}^\dagger a_{in} \rangle$ . With the mean-field approximation for this input noise, it follows that the optical mode is modulated by an extra amount of  $\sqrt{\kappa} a_{in}$ . The input-output relation is then obtained as :

$$a_{out} = a_{in} - \sqrt{\kappa} a, \quad (2.19)$$

where  $a_{out}$  is the measured field exiting the cavity through the mirror. Equations of Motion, Using the Hamiltonian of the system derived in Equation (2.19) and the expressions for fluctuations and dissipations, we obtain the quantum Langevin equations (QLEs) corresponding to the cavity field and the mechanical mode as

$$\dot{a} = \frac{1}{i\hbar} [a, H_{sys}] - \frac{\kappa}{2} a + \sqrt{\kappa} a_{in} = -\frac{\kappa}{2} a + i\Delta_0 a + ig_0 (b^\dagger + b) a + A_l + \sqrt{\kappa} a_{in} \quad (2.20)$$

$$\dot{b} = \frac{1}{i\hbar} [b, H_{sys}] - \frac{\gamma}{2} b + \sqrt{\gamma} b_{in} = -\frac{\gamma}{2} b + i\omega_m b + ig_0 a^\dagger a + \sqrt{\gamma} b_{in}. \quad (2.21)$$

# CHAPTER THREE

## 3 MODEL AND METHODOLOGY

In this thesis work, we would analyze the coupled an optomechanical systems for sensing applications: displacement and mass, using the following methods. The progress of precision sensing applications using cavity optomechanical systems would be analyzed. The evaluation is organized in the following way. Introducing the physical principles of optomechanical sensing, including a discussion of the noises and sensitivity of the physical principles of systems of optomechanical coupling, optomechanical sensing platforms, noise, sensitivity and bandwidth.

We would review the progress in displacement sensing and mass sensing. we would use the cavity mode coupled with a mechanical mode, which can be modeled as a FabryPerot ( F – P ) cavity with one fixed mirror and one movable mirror mounted on a spring. To precisely model the coupling process, we would provide a full quantum description of cavity optomechanical interaction form of the below. A general cavity optomechanical system with the total Hamiltonian:

$$\hat{H} = \hat{H}_0 + \hat{H}_{int} + \hat{H}_{dri} \quad (3.1)$$

From the above equation we would use the following terms:

- The optical driving of the system  $\left(\hat{H}_{dri}\right)$ ,
- The Hamiltonian of the uncoupled optical and mechanical modes,  $\left(\hat{H}_0\right)$   
and

- The optomechanical interaction between the optical mode and the mechanical mode,  $\left(\widehat{H}_{int}\right)$
- Would provide for the optomechanical system an excellent platform for precision displacement sensing and mass sensing. To measure the displacement  $x(\omega)$  of the mechanical resonator in the frequency domain by the displacement sensing based on the optomechanical system.
- The resonator acts as a mass sensor due to the resonance frequency sensitivity of the mass absorbed onto it.

### **3.1 Source of data**

The researcher has used secondary sources of data for this study. The secondary sources of information include; books, journals, and internet sources.

### **3.2 Methods of data analysis and interpretation**

To achieve this study's desired objective was applied the investigator used Mathematical waveform to draw the graphs for Resonance frequency versus effective mass.

# CHAPTER FOUR

## 4 RESULTS AND DISCUSSIONS

### 4.1 Cavity optomechanical coupling

A Fabry-Perot cavity with a compliant end mirror make up the canonical cavity optomechanical system. By observing the phase or amplitude of the field seeping out of the cavity, we would be interested in measuring the displacement of the end-mirror. In order to achieve this, we give a fundamental explanation of the dynamics of the cavity optomechanical system, which is effectively characterized by a pair of linked differential equations (Aspelmeyer et al., 2014).

$$m\ddot{x}(t) + m\gamma_m\dot{x}(t) + m\omega_m^2x(t) = F_{ext}(t) + F_{RP}(x,t) \quad (4.1)$$

$$\dot{a}_c(t) = \left( \kappa + i(\omega_0 - \omega_c(x(t))) \right) a_c(t) + \sqrt{2k_1}E_{in}(t) \quad (4.2)$$

Equation (4.1) is the equation of motion for a damped harmonic oscillator and describes the displacement,  $x$ , of the compliant end mirror. The variables  $m$ ,  $\gamma_m$ , and  $\omega_m$  are the effective mass, damping rate, and natural frequency of the mirror, respectively.  $F_{ext}(t)$  is the sum of all the external forces on the mirror, excluding the radiation pressure force,  $F_{RP}(x,t)$ , which we shall return to shortly.

Equation (4.2) is the equation of motion for the complex amplitude,  $a_c$ , of the intracavity field in the slowly varying envelope approximation, normalized so that  $|a_c(t)|^2$  is the energy of the intracavity field. The variables  $\kappa$ ,  $\omega_0$ ,  $\omega_c$ ,  $k_1$ , and  $E_{in}$  denote the energy decay rate of the cavity, the angular

frequency of the input field, the resonance frequency of the cavity, the rate of energy decay through the input mirror, and the slowly varying complex amplitude of the input field, respectively.

The two equations are coupled via the position-dependent cavity resonance frequency:

$$\omega_c(t) = \frac{mc}{2(L+x)} \approx \omega_c(0) - Gx \quad (4.3)$$

Where  $m$  is the cavity mode order (an integer value),  $c$  is the speed of light,  $L$  is the nominal cavity length, and:

$$G = \frac{d\omega_c}{dx} \approx \frac{\omega_c}{L} \quad (4.4)$$

Is the optomechanical coupling factor. Note that for  $x = 0$ , the radiation force experienced by the mirror can be expressed in a number of different ways. This would help us understand how  $G$  influences the dynamics of the mechanical resonator.

$$F_{RP}(x,t) = \frac{2P_{\text{Circ}}(t)}{c} = \frac{|a_c(t)|^2}{L} = \frac{\hbar\omega_c n_c}{L} = \hbar G n_c = \frac{G|a_c(t)|^2}{\omega_c} \quad (4.5)$$

Where  $P_{\text{Circ}}(t)$  is the energy that is flowing through the optical cavity (the energy that strikes the end mirror) and  $n_c$  is the intracavity photon number (Mason, Chen, Rossi, Tsaturyan, & Schliesser, 2019).

Linearizing about small fluctuations,  $a_c - \langle a_c \rangle$  and making the notational change:  $a_c \rightarrow \langle a_c \rangle + a_c$ ,  $F_{RP} \rightarrow \langle F_{RP} \rangle + F_{RP}$ , etc., yields the linearized cavity optomechanical equations (assuming a stationary input field,  $E_{in} = 0$ ):

$$m\ddot{x}(t) + m\gamma_m\dot{x}(t) + m\omega_m^2 x(t) = F_{ext}(t) + F_{RP}(x,t) \quad (4.6)$$

$$\dot{a}_c(t) = (\kappa + i\Delta)a_c(t) + iGx(t) \langle a_c \rangle \quad (4.7)$$

Cavity field, and  $F_c(x, t) = \frac{G(\langle a_c \rangle a_c + c.c.)}{\omega_c}$  is the fluctuating radiation pressure force. The left side of equation (4.7) can be omitted if the cavity field reacts to mechanical motion instantly (the so-called "bad cavity limit"), in which case the following is the sensitivity of the intracavity field to mechanical motion:

$$a_c(t) \approx \frac{iGx(t)}{k + i\Delta} \langle a_c(t) \rangle \quad (4.8)$$

The input-output relation can also connect the output field to the circulating field (Aspelmeyer et al., 2014):

$$E_{\text{out}}(t) = \sqrt{2k_2} a_c(t) \quad (4.9)$$

We observe that, in the most basic situation of a cavity that is resonantly probed ( $\Delta = 0$ ) the output field's phase,  $\phi_{\text{out}}(t)$ , the following familiar expression can be made in terms of the location of the mechanical resonator:

$$\phi_{\text{out}}(t) \approx \frac{\text{Im}[E_{\text{out}}(t)]}{\text{Re}[E_{\text{out}}(t)]} = \frac{\text{Im}[a_c(t)]}{\text{Re}[a_c(t)]} = \frac{Gx(t)}{k} = \frac{\omega_c x(t)}{Lk} = \frac{2Fx(t)}{\lambda} \quad (4.10)$$

where  $F = \frac{\pi c}{L\kappa}$  is the cavity finesse.

Taking the Fourier transform of both sides of the equation as a whole, we can relate  $a_c$  and  $x$  vulnerability for optomechanical systems:

$$X_{a_c(x)}(\omega) = \frac{a_c(\omega)}{X(\omega)} = \frac{G}{\Delta + \omega - ik} \quad (4.11)$$

Where the finite cavity response term has been kept around for completeness.  $\dot{a}_c(t)$ , in equation (4.11), leading to the term,  $\omega$ , a negligible amount in the bad cavity limit's denominator,  $\omega \ll \kappa$ .

## 4.2 Physical principles of cavity optomechanical sensing

The physical foundations of cavity optomechanical sensing are presented in this subdivision. We first examine the optomechanical coupling principle, then go over the various platforms for optomechanical sensing, before talking about the noise, sensitivity, and bandwidth of optomechanical sensing.

### 4.2.1 Optomechanical coupling

The term "optomechanical coupling" refers to the interaction of an optical radiation field with a mechanical vibrational mode. We use a  $F - P$  optomechanical system as an illustration (Figure 4). The movable mirror experiences radiation pressure from the cavity fields, which modifies the mechanical modes as well as the resonance frequency and damping rate. The position of the movable mirror is modulated by the mechanical vibration of the spring at the same time, changing the cavity length  $L$  and optical resonant frequency  $\omega_c$ . We present a complete quantum theory of cavity optomechanical coupling below to accurately characterize the coupling process. Here the cavity field and the mechanical resonator are represented as quantized bosonic fields. Quantum Langevin equations can be used to explain the system dynamics, starting with the system Hamiltonian and taking into account dissipation. Consider the standard cavity optomechanical system (Figure 4), which consists of a single optical cavity mode linked with a mechanical mode and may be described as a  $F - P$  cavity with one fixed mirror and one moveable mirror positioned on a spring.

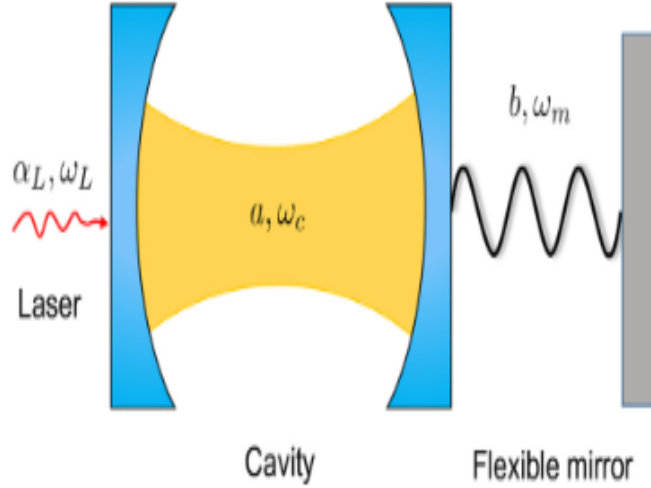


Figure 4.1: Schematic of general optomechanical system (Zhu et al., 2020)

Schematic of a typical optomechanical system with a laser-driven Fabry-Perot (F – P) cavity is shown in Figure (4). The right mirror can be moved, but the left mirror is fixed.

### 4.3 Cavity of optomechanical systems

Figure 5 below shows a cavity with two mirrors, one of which is fixed and the other of which is movable (say  $L$ ) and powered by a laser source (such as one that has amplitude  $A_l$  and frequency  $\omega_l$ ). The laser’s injected light field is amplified by the cavity, and the intracavity photons’ radiation pressure force causes a constant transfer of momentum to the end mirror’s mechanically compliant surface. Consequently, it is possible to estimate that the mechanical mirror oscillates at a resonance frequency of  $\omega_m$  and an effective mass of  $m$ . Then, one can concentrate on the cavity’s single optical mode (let’s assume with frequency  $\omega_o = n\pi c/L$  with mode number  $n$ ) This is close to the frequency of a laser. In relation to a mechanical displacement of  $\hat{x}$ , the new cavity resonance frequency can be expressed as (Kampel et al., 2017)

$$\omega_c(\hat{x}) = \frac{n\pi c}{L + \hat{x}} = \frac{\omega_o}{1 + \hat{x}/L} \approx \omega_o \left(1 - \frac{\hat{x}}{L}\right) \quad (4.12)$$

We may derive the frequency pull parameter from this relationship.  $G = \partial\omega_c(\hat{x})/\partial\hat{x} = \omega_o/L$ , It measures the linear dispersive shift of the optical field's resonance frequency brought about by the mechanics. The cavity's typical resonant frequency ( $\omega_o$ ) is on the order of  $10^{15}$  Hz, as opposed to the mechanical resonance frequency ( $\omega_m$ ) is usually in the range of MHz to GHz. Therefore, the laser frequency is modified so that its detuning  $\Delta_0$  is comparable to  $\omega_m$ . The influence of thermal photons entering the cavity can also be safely ignored because of how high the optical frequencies are, and the cavity can be thought of as being connected with a reservoir that is always at absolute zero. The laser drive does, however, add noise to the system. On the other hand, the quantum dynamics of the mechanical mode are significantly influenced by thermal phonons (Eichenfield et al., 2009).

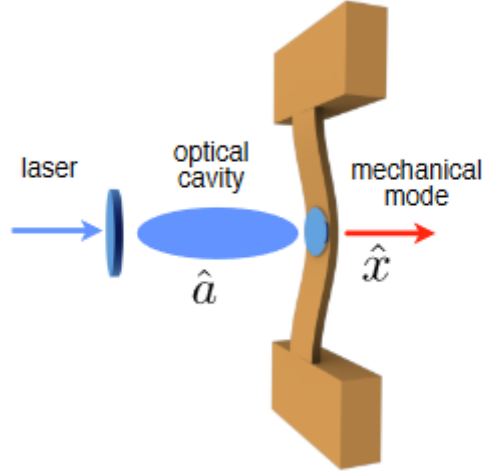


Figure 4.2: A laser-driven cavity  
(Eichenfield et al., 2009)

$$\hat{H} = \hbar\omega(\hat{x})\hat{a}^\dagger\hat{a} + \hbar\Omega\hat{b}^\dagger\hat{b} + \dots \quad (4.13)$$

The words represented by the dots, such as "laser driving," "photon," and "phonon decay," would be considered later. Here  $\hat{x} = x_{\text{ZPF}}(\hat{b} + \hat{b}^\dagger)$  is the displacement of the single vibrational mode that we choose to focus on (for the time being, ignoring all other normal modes). Here  $x_{\text{ZPF}} = (\hbar/2m\Omega)^{1/2}$  is the width of the mechanical ground state wave function, or the size of the mechanical zero-point fluctuations. The optical resonance frequency can be expanded in the displacement, with  $L$  the cavity length and  $\omega_{\text{cav}}$  the optical resonance frequency for  $x = 0$  (Guggenheim et al., 2017):

$$\omega(\hat{x}) = \omega_{\text{cav}} \left( 1 - \frac{\hat{x}}{L} + \dots \right) \quad (4.14)$$

Thus,  $\hat{H}$  contains a term of the type  $-\hat{F}\hat{x}$ , where we identify  $\hat{F}$  as the radiation pressure force:

$$\hat{F} = \frac{\hbar\omega_{\text{cav}}}{L}\hat{a}^\dagger\hat{a} \quad (4.15)$$

The conventional optomechanical Hamiltonian can be summarized as follows:

$$\hat{F} = \frac{\hbar\omega_{\text{cav}}}{L}\hat{a}^\dagger\hat{a} \quad (4.16)$$

Here we have identified

$$g_0 = \omega_{\text{cav}}\frac{x_{\text{ZPF}}}{L} \quad (4.17)$$

as the optomechanical single-photon coupling strength. It represents the optical frequency shift produced by a zero-point displacement. Therefore, the optomechanical coupling term to be used in the following is:

$$-\hbar g_0\hat{a}^\dagger\hat{a}(\hat{b} + \hat{b}^\dagger) \quad (4.18)$$

In essence, we are facing a parametric coupling where a mechanical displacement controls the frequency of some (driven) resonance. It is no surprise, then, that this is a rather generic situation which can be obtained in many different setups. By means of this investigation, we take into account Hamiltonian( $\hat{H}$ ) system written as (Aspelmeyer et al., 2014):

$$\hat{H} = \hat{H}_0 + \hat{H}_{\text{int}} + \hat{H}_{\text{dri}} \quad (4.19)$$

Here the first term of equation (4.19),  $H_{\text{drive}}$  describes the optical driving of the system. Consider that a continuous-wave laser is injected into the system, and the laser. Hamiltonian is written as:

$$\hat{H}_{\text{drive}} = \Omega^* e^{i\omega_L t}\hat{a} + \Omega e^{-i\omega_L t}\hat{a}^\dagger \quad (4.20)$$

Where  $\omega_L$  is the input laser frequency,  $\hat{a}(\hat{a}^\dagger)$  is the bosonic annihilation (creation) operator of the cavity optical mode, and

$$\Omega = \sqrt{\frac{k_{ex}P}{\hbar\omega_L}} e^{i\phi} \quad (4.21)$$

denotes the driving strength, where  $P$  is the input laser power,  $\phi$  is the initial phase of the input laser, and  $k_{ex}$  is the decay rate of input-cavity coupling. The second term of equation (4.19),  $\hat{H}_{\text{free}}$  is the Hamiltonian of the uncoupled optical and mechanical modes, described by

$$\hat{H}_{\text{free}} = \omega_c \hat{a}^\dagger \hat{a} + \omega_m \hat{b}^\dagger \hat{b} \quad (4.22)$$

Among it, the mechanical mode is regarded as a quantum harmonic oscillator, where  $\hat{b}(\hat{b}^\dagger)$  is the bosonic annihilation (creation) operator of the mechanical modes, and  $\omega_c$  ( $\omega_m$ ) is the optical (mechanical) resonance frequency. The commutation relations satisfy  $[\hat{b}, \hat{b}^\dagger] = 1$  and  $[\hat{a}, \hat{a}^\dagger] = 1$ . The displacement operator of the mechanical mode is given by

$$\hat{X} = ZPF(\hat{b} + \hat{b}^\dagger) \quad (4.23)$$

where

$$X_{ZPF} = \sqrt{\frac{\hbar}{2m_{eff}\omega_m}} \quad (4.24)$$

is the zero-point fluctuation, with  $m_{eff}$  being the effective mass of the mechanical mode.

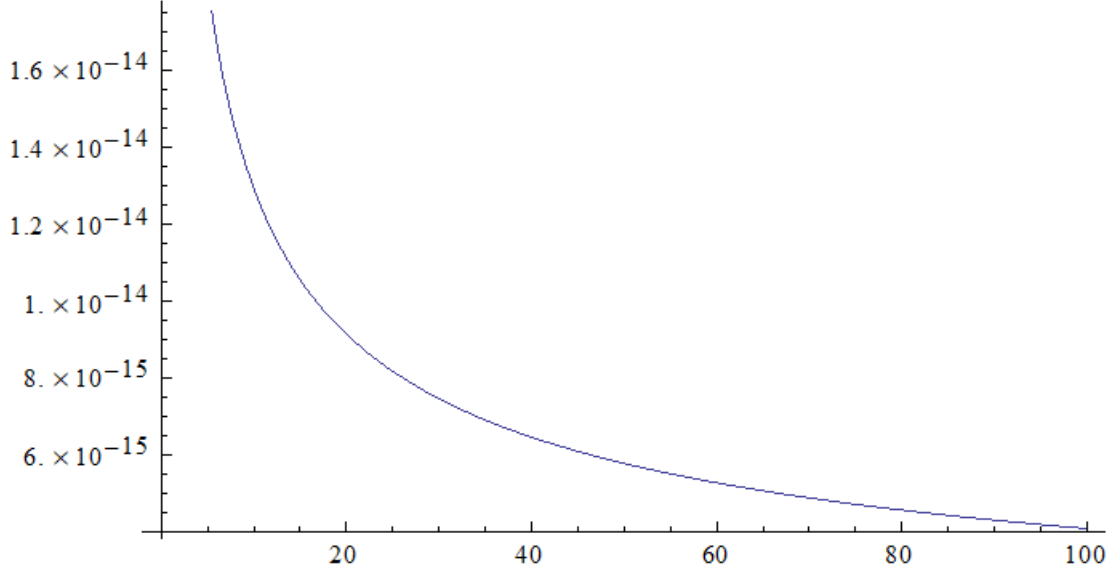


Figure 4.3: Zero point fluctuation ( $X_{ZPF}$ ) versus the optical (mechanical) resonance frequency ( $\omega_m$ )

The above figure 4.3 shows as the zero-point fluctuation, with  $m_{eff}$  increase optical (mechanical) resonance frequency ( $\omega_m$ ) is decrease. The third term of equation (4.19),  $H_{int}$  describes the optomechanical interaction between the optical mode and the mechanical mode, which is given by

$$\hat{H}_{int} = g\hat{a}^\dagger\hat{a}(\hat{b} + \hat{b}^\dagger) \quad (4.25)$$

Here

$$g = x_{ZPF}G \quad (4.26)$$

Represents the single photon optomechanical coupling strength, and

$$G = \frac{\partial \omega_c(x)}{\partial x} \quad (4.27)$$

is the strength of the optomechanical connection corresponding to the optical frequency shift per displacement. Simply assuming that the mechanical resonator's position modulates the optical resonance frequency and apply-

ing Taylor expansion at the original point to obtain this Hamiltonian, which is denoted by

$$\omega_c(x) = \omega_c(0) + x \frac{\partial \omega_c(x)}{\partial x} + O(x) \simeq \omega_c(0) + g(\hat{b}^\dagger + \hat{b}) \quad (4.28)$$

Law's study provides a more exact and thorough derivation of this Hamiltonian (Law, 1995).

Additionally, the radiation pressure force is expressed as the derivation of  $H_{\text{int}}$  relating to displacement:

$$F = -\frac{dH_{\text{int}}}{dx} = \frac{g}{xZPF} \hat{a}^\dagger \hat{a} \quad (4.29)$$

The system Hamiltonian is changed to in the frame revolving at the input laser frequency  $\omega_L$ .

$$\hat{H} = -\Delta \hat{a}^\dagger \hat{a} + \omega_m \hat{b}^\dagger \hat{b} + g \hat{a}^\dagger \hat{a} (\hat{b}^\dagger + \hat{b}) + \left( \Omega^* \hat{a} + \Omega \hat{a}^\dagger \right) \quad (4.30)$$

$$\Delta = \omega_L - \omega_c \quad (4.31)$$

is the input-cavity detuning. The quantum Langevin equations are provided by

$$\dot{\hat{a}} = \left( i\Delta - \frac{k}{2} \right) \hat{a} - ig \hat{a} (\hat{b}^\dagger + \hat{b}) - i\Omega - \sqrt{k_{\text{ex}}} \hat{a}_{\text{in},\text{ex}} - \sqrt{k_0} \hat{a}_{\text{in},0} \quad (4.32)$$

$$\dot{\hat{b}} = \left( -i\omega_m - \frac{\gamma}{2} \right) \hat{b} - ig \hat{a}^\dagger \hat{a} - \sqrt{\gamma} \hat{b}_{\text{in}} \quad (4.33)$$

Here  $k_0$  is the intrinsic cavity decay rate,

$$\mathcal{K} = \mathcal{K}_0 + \mathcal{K}_{\text{ex}} \quad (4.34)$$

is the total cavity decay rate,  $\gamma$  is the damping rate of the mechanical mode. Besides,  $\hat{a}_{in,0}$ ,  $\hat{a}_{in,ex}$ , and  $\hat{b}_{in}$  are the noise operators associated with the intrinsic cavity decay, external cavity decay(input-cavity coupling), and mechanical damping. Additionally, the quality factors of optical and mechanical mode are defined as the ratio of their resonant frequency and damping rate respectively, which are  $Q_0 = \frac{\kappa}{\omega_c}$  and  $Q_m = \frac{\gamma}{\omega_m}$  (Arcizet et al., 2006).

## 4.4 Optomechanical sensing applications

### 4.4.1 Noise, sensitivity, and bandwidth

Sensitivity, which is measured by the noise equivalent signal and is defined as the smallest detectable signal, is the most crucial factor for optomechanical sensors. As a result, we would talk about the noise sources in optomechanical systems, which include probing laser noise and thermal noise from an environment with a non-zero temperature. The probe lasers' noise consists of both conventional and quantum noise. Laser intensity noise, phase noise, and other types of classical noise (mostly in the low frequency range) can be reduced using some measuring techniques. The quantum property of photons, often known as photon shot noise, is the source of the noise (Anetsberger et al., 2009).

The imprecision noise and the back action noise are both parts of the quantum noise in optomechanical systems. The back action noise (also known as radiation pressure noise) results from the radiation pressure force of photons acting on the mechanical resonator, whereas the imprecision noise is caused by the uncertainty connection between the number and phase of photons. Since the probe laser power is typically low in optomechanical sensing applications to prevent thermally induced optical resonance shift, back action

noise is typically minimal. Therefore, in this section, we focus primarily on the thermal noise and shot noise in optomechanical sensing systems (Hoff et al., 2013).

A physical quantity's sensitivity at the frequency  $\omega$  (such as displacement, force, etc.) is governed by

$$\sqrt{S_{AA}(\omega)} \quad (4.35)$$

where the noise spectrum is defined as the Fourier transform of the autocorrelation function of  $A$  as

$$S_{AA}(\omega) = \int_{-\infty}^{+\infty} \langle A(t)A(0) \rangle e^{i\omega t} dt \quad (4.36)$$

Because of their high optical quality factors and wide mechanical resonance frequency range, micro toroids make an excellent platform for high sensitivity and wide bandwidth optomechanical sensing (Aspelmeyer et al., 2014).

$$F_{th} = \sqrt{S_{FF}(\omega)} = \sqrt{2m_{eff}\gamma k_B T} \quad (4.37)$$

The displacement noise power spectrum of a mechanical resonator from thermal noise is

$$S_{xx}^{th} = |X(\omega)|^2 S_{th}^2 = \frac{2\gamma k_B T}{m_{eff} [(\omega_m^2 - \omega^2)^2 - \omega^2 \gamma^2]} \quad (4.38)$$

Contrarily, the power spectrum of the displacement noise resulting from the laser shot noise is frequency independent (Bowen & Milburn, 2015).

$$S_{xx}^{th} = \frac{k}{16\eta N g_0^2} \left( 1 + \frac{\omega^2}{k^2} \right) \quad (4.39)$$

Where  $\kappa$  is the total optical decay rate,  $\eta = 0.8$  is the optical detection efficiency,  $N$  is the intra cavity photon number

$$N = \frac{P\kappa_{ex}}{\hbar\omega_L} \quad (4.40)$$

For probe power  $P$ . In addition to the probe power, the optical  $Q_0$  factor and optomechanical coupling strength also affect the sensitivity. The higher  $Q_0$  and  $G$  are, the higher the readout sensitivity is, and therefore it is easier to reach thermal noise limited sensitivity (Arcizet et al., 2006).

To measure an external physical quantity, practical optomechanical sensors are typically used. For instance, in optomechanical systems for magnetic field detection, it is simply detecting a magnetic field ( $B$ ) induced force ( $F \propto B$ ) influencing the mechanical resonator. Consequently, we would discuss a mechanical resonator's force sensitivity in the sections that follow (Arcizet et al., 2006).

$$\sqrt{S_{FF}(\omega)} = \frac{\sqrt{S_{FF}(\omega)}}{|X(\omega)|} \quad (4.41)$$

It is clear that the mechanical resonance frequency is where force sensitivity peaks, and that as frequency decreases away from the mechanical resonance, the sensitivity increases. Reaching a thermal noise restricted domain is hence advantageous for greater sensitivity. The mechanical damping rate  $\gamma$  affects the peak sensitivity. In order to improve force sensitivity, it is crucial to have a lower damping rate (or larger mechanical  $Q_m$  factor).

In order to characterize the frequency dependent sensitivity, we define a bandwidth as the frequency range in which the sensitivity is better than twice of the peak sensitivity. It can be seen that, the higher the probe power is, the broader the bandwidth is. In addition to probe power, having a higher optical

factor  $Q_0$  and optomechanical coupling strength are also helpful to reach the thermal noise limited sensitivity, and therefore broadening the bandwidth of the optomechanical force sensor. This means that optomechanical sensors provide an ultrabroad band sensing platform, with band width easily reaching tens of MHz (Kampel et al., 2017).

#### 4.4.2 Displacement sensitivity

By monitoring the calibrated displacement noise spectra as a function of power, it is possible to observe both the damping rate and the effective temperature of a mechanical mode. Optical interferometers are among the most sensitive displacement sensors, in fact. The shot noise limited displacement sensitivity of a cavity opto-mechanical system for quantum-noise-limited is given by (Galeotti et al., 2020):

$$\delta x_{\min} \cong \lambda / \left( 8\pi F \sqrt{\frac{\eta P}{\hbar \omega}} \right) \quad (4.42)$$

The foundation of accurate detection of several physical characteristics, including force, magnetic field, acceleration, and ultrasound, is ultrahigh-sensitivity displacement sensing. An ideal platform for precise displacement sensing is provided by the optomechanical system. The mechanical resonator's displacement  $x(\omega)$  in the frequency domain is measured using displacement sensing based on an optomechanical system. The optical transmission signal typically contains information about the mechanical resonator's displacement thanks to optomechanical coupling. A photo detector and optical spectrum analyzers can then read it out (Hu, Xiao, Liu, & Gong, 2013).

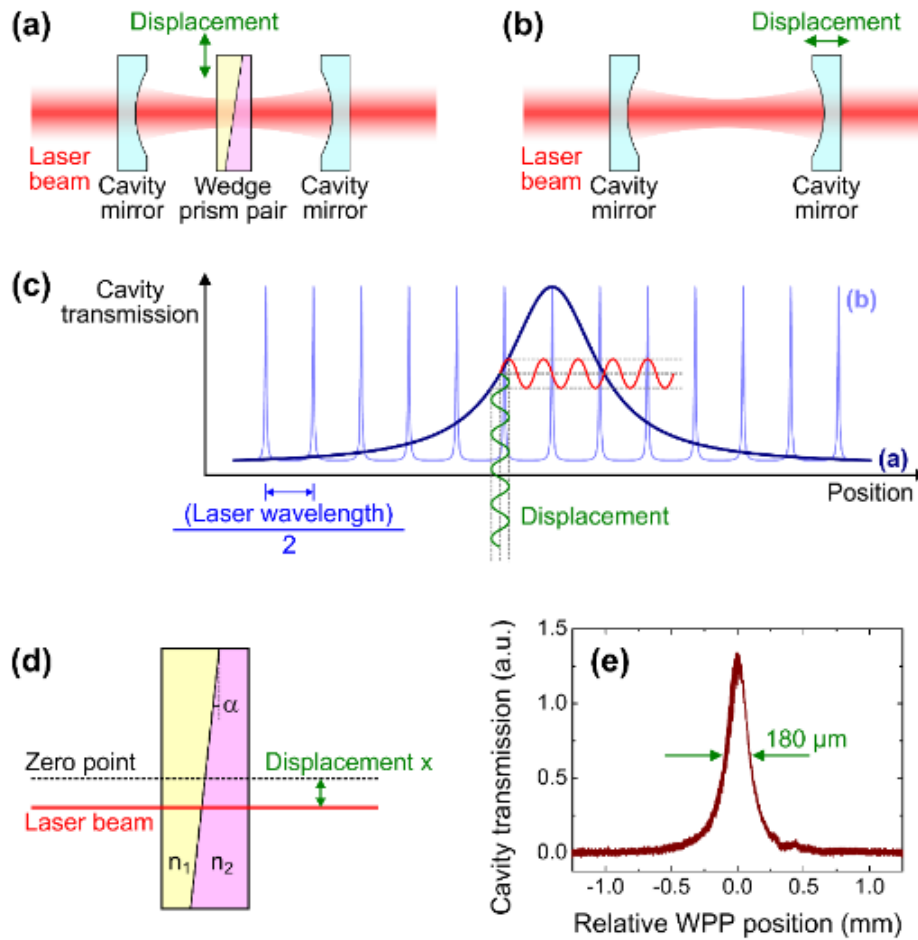


Figure 4.4: Optical-cavity-based displacement sensor (Tadigadapa & Mateti, 2009)

Schematic representation of our displacement sensor, which consists of an intra-cavity wedge prism pair (WPP) and a free-space Fabry-Pérot optical cavity. While the two cavity mirrors and the laser frequency are fixed, the change in cavity transmission results from the WPP movement in the direction perpendicular to the laser beam. (b) A diagram of a possible simplified alternative optical-cavity-based displacement sensor, in which the displacement of a cavity mirror along the laser beam direction is sensed. (c) Cavity transmission with fixed laser frequency as a function of WPP position (for (a), dark blue curve) or cavity mirror position (for (b), light blue curve). In contrast to instance (b), where the range of observable displacement is constrained to the half of the laser wavelength due to the  $2\pi$  ambiguity,

employment of a WPP in case (a) offers a far wider range of measured displacement.

A red curve is also shown to show how the variation in cavity transmission for the instance of (a) changes with the change in WPP displacement.

(d) The WPP structure in combination with a transmitting laser beam that is offset from the zero point by a distance of  $x$ .  $n_1$  and  $n_2$  are the refractive indices of the two wedge prisms, and  $\alpha$  is the common wedge angle.

(e) Measured cavity transmission of a manufactured WPP across a range of positions. The complete width at half maximum of a resonance shows a definite peak of  $180\mu\text{m}$ , which is far greater than the measurement's laser wavelength, which was  $775\text{ nm}$  (Tallur & Bhave, 2013).

#### **4.4.3 Mass sensitivity**

The mass-to-charge ratio of analytes is measured using conventional mass spectrometers, which can also resolve individual atoms. They are nonetheless limited in their ability to weigh heavier analytes like colloidal and aerosol particles that are larger than a few atoms due to their poor dynamic range. Such particles are frequently fragmented using dissociation techniques, which modifies their natural properties and makes it difficult to recover the mass spectrum. Optomechanical sensors, on the other hand, use a mechanical mechanism like a micro-resonator to monitor the optical motion that it transmits in order to measure mass (Aebersold & Mann, 2003).

The mass spectrometer has historically been used to determine the mass of small charged particles (Griffiths, 2008). The measurement of the mass of electrically neutral particles has emerged as a hot research topic as a result of the requirement for the measurement of molecular-scale substances in the

domains of chemistry and biology (Li & Zhu, 2013).

Microresonators are typically particularly sensitive to the disruption brought on by external impact due to their tiny size. These mechanical resonator-based mass sensors do not require the molecular ionization process; instead, they rely on a resonant frequency change caused by an accreted mass, opening a new chapter in the measurement of nanotechnology (Chaste et al., 2012).

Because the resonance frequency sensitivity of the mass absorbed onto the resonator, it functions as a mass sensor. Mass sensing's basic idea is still straightforward, despite the measurement method's complexity. Harmonic oscillators, which have an effective mass, can be used to describe mechanical resonators  $m_{eff}$ , a spring constant  $k$ , and a resonance frequency (Yu, Jiang, Lin, & Lu, 2016).

$$f_m = \frac{1}{2\pi} \sqrt{\frac{k}{m_{eff}}} \quad (4.43)$$

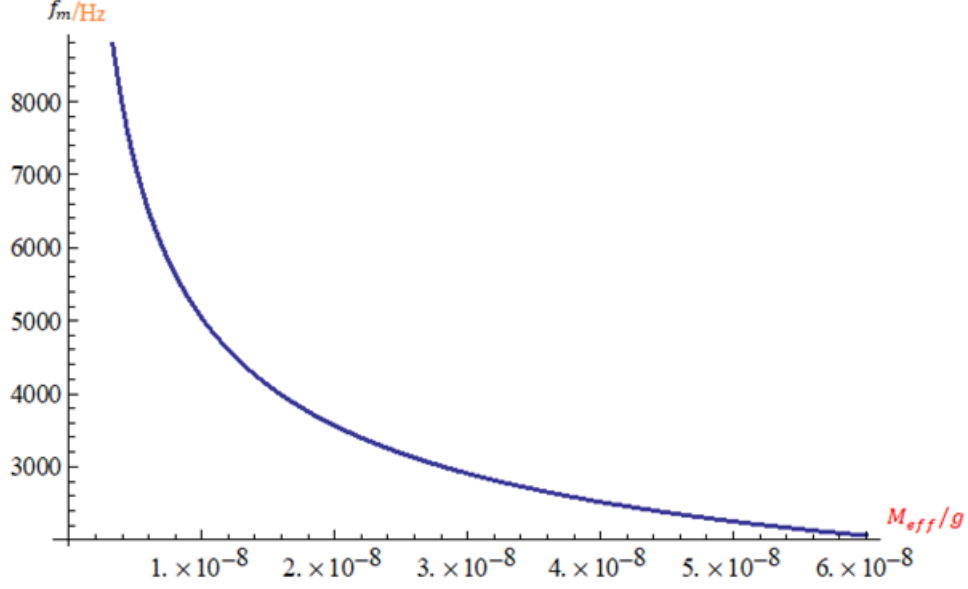


Figure 4.5: Resonance frequency versus effective mass ( $m_{eff}$ )

The above figure 4.5 shows as resonance frequency  $f_m$  increase effective mass  $m_{eff}$  decrease. The spring's effective mass has changed  $m_{eff}$  will result in the mechanical resonator's resonance frequency shifting  $f_m$ , which is

$$m_{eff}\delta = \frac{2m_{eff}\delta f_m}{f_m} \quad (4.44)$$

In terms of quality, we simply think of the deposited bulk as

$$m_d \approx m_{eff}\delta = \frac{-2m_{eff}\delta}{f_m} f_m \quad (4.45)$$

Equation (4.45) shows how smaller masses can be detected when the instrument's resolution is fixed by lowering the mechanical resonator's effective mass or raising its resonance frequency. Particularly, the binding position of the object in the mechanical mode affects the relationship between deposition mass and the frequency shift (Dohn, Svendsen, Boisen, & Hansen, 2007).

Cavity optomechanical systems with a mass sensor owing to the resonator's

$6.0 \times 10^{-8}$  g effective mass and  $1.5 \times 10^3$  The system showed 150 fg for the effective mechanical quality factor ( $1.5 \times 10^{-13}$  g) mass sensitivity.

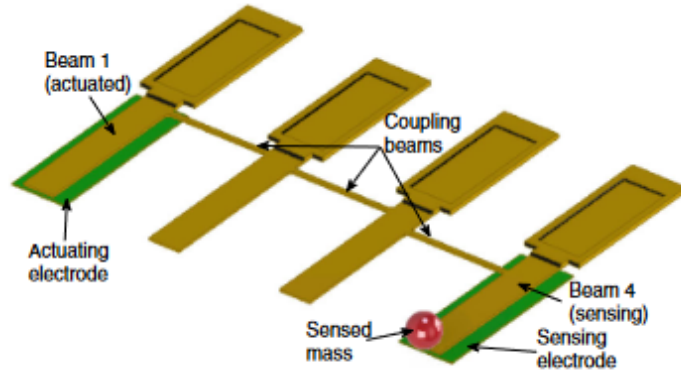


Figure 4.6: Schematic of the mass sensor made of an array of mechanically coupled microcantilever beams

(Rabenimanana et al., 2019)

Figure (4.6) shows an array of four coupled microbeams, with slightly varying lengths and different coupling locations. A small mass is added to the tip of the fourth beam, which constitutes the sensing resonator. The first beam is selected to be the actuated resonator. The sensing beam is assumed to be coated with a detector material with high affinity to the mass to be characterized. In the present sensor model, we consider the first beam to be the actuating one and allow for arbitrary selection of the sensing resonator, which affect the sensitivity of the device.

# CHAPTER FIVE

## 5 CONCLUSIONS AND RECOMMENDATIONS

### 5.1 Conclusions

When light and mechanical resonators interact in cavity optomechanical systems, it is possible to explore macroscopic quantum systems theoretically and to use precise sensing in practical applications. “Cavity-optomechanics” aims to study the quantum properties of mechanical systems. The cavity optomechanical systems’ resonant enhancement of both mechanical and optical response has enabled precision sensing of a wide range of physical quantities, including displacements, masses, forces, accelerations, magnetic fields, and ultrasounds. The usage of cavity optomechanical systems for applications involving precise sensing is examined in this research. The spring’s effective mass has changed  $m_{eff}$  will result in the mechanical resonator’s resonance frequency shifting. Optomechanical mass sensors provide a non-destructive and direct pathway to probe the mass of particles in various forms. Following is the breakdown of the presentation: We would first go through the fundamentals of optomechanical sensing, along with a discussion of the noises and sensitivity of the systems, and then would look at developments in mass and displacement sensing. We end by offering a verdict and a projection.

## 5.2 Recommendations for future work

This study has several strengths and limitations that should be addressed to study Cavity coupled optomechanical systems for sensing applications: Displacement and Mass. Therefore, to describe Cavity coupled optomechanical systems for sensing applications: Displacement and Mass, a more systematic study (similar research) that can address the point in limitations; still there are no enough literatures. Future researchers should deeply investigate on:

- Cavity coupled optomechanical systems for sensing applications: Displacement and Mass by achieve more robust, portable, and cheaper devices, integration of the lasers and photo detectors with the on-chip optomechanical sensors are required in the future.
- Cavity coupled optomechanical systems for sensing applications: Displacement and Mass by analyzing With their superior sensitivity, broad bandwidth, low power consumption, on-chip integration capability, and high technology readiness, moving forward, researchers will believe these precision optomechanical sensors will find applications in real world in the near future.
- Cavity coupled optomechanical systems for sensing applications: Displacement and Mass by serving as the field of optomechanics shall serve as a versa-tile platform to propose, interpret or predict quantum phenomena as well as advance quantum technologies in the years to come.

## References

- Aebersold, R., & Mann, M. (2003). Mass spectrometry-based proteomics. *Nature*, 422(6928), 198–207.
- Akram, U., Munro, W., Nemoto, K., & Milburn, G. (2012). Photon-phonon entanglement in coupled optomechanical arrays. *Physical Review A*, 86(4), 042306.
- Aldana, S., Bruder, C., & Nunnenkamp, A. (2013). Equivalence between an optomechanical system and a kerr medium. *Physical Review A*, 88(4), 043826.
- Allain, P. E., Schwab, L., Mismar, C., Gely, M., Mairiaux, E., Hermouet, M., ... others (2020). Optomechanical resonating probe for very high frequency sensing of atomic forces. *Nanoscale*, 12(5), 2939–2945.
- Alnis, J., Matveev, A., Kolachevsky, N., Udem, T., & Hänsch, T. (2008). Subhertz linewidth diode lasers by stabilization to vibrationally and thermally compensated ultralow-expansion glass fabry-pérot cavities. *Physical Review A*, 77(5), 053809.
- Anetsberger, G., Arcizet, O., Unterreithmeier, Q. P., Rivière, R., Schliesser, A., Weig, E. M., ... Kippenberg, T. J. (2009). Near-field cavity optomechanics with nanomechanical oscillators. *Nature Physics*, 5(12), 909–914.
- Arcizet, O., Cohadon, P.-F., Briant, T., Pinard, M., Heidmann, A., Mackowski, J.-M., ... Rousseau, L. (2006). High-sensitivity optical monitoring of a micromechanical resonator with a quantum-limited optomechanical sensor. *Physical review letters*, 97(13), 133601.
- Aspelmeyer, M., Kippenberg, T. J., & Marquardt, F. (2014). Cavity optomechanics. *Reviews of Modern Physics*, 86(4), 1391.
- Bai, C.-H., Wang, D.-Y., Zhang, S., Liu, S., & Wang, H.-F. (2020). Strong mechanical squeezing in a standard optomechanical system by pump modulation. *Physical Review A*, 101(5), 053836.
- Binnig, G., Gerber, C., Stoll, E., Albrecht, T., & Quate, C. (1987). Atomic resolution with atomic force microscope. *Europhysics Letters*, 3(12), 1281.
- Bowen, W. P., & Milburn, G. J. (2015). *Quantum optomechanics*. CRC press.
- Chae, J., An, S., Ramer, G., Stavila, V., Holland, G., Yoon, Y., ... Centrone, A. (2017). Nanophotonic atomic force microscope transducers enable chemical composition and thermal conductivity measurements at the nanoscale. *Nano letters*, 17(9), 5587–

5594.

- Chan, J., Alegre, T. M., Safavi-Naeini, A. H., Hill, J. T., Krause, A., Gröblacher, S., ... Painter, O. (2011). Laser cooling of a nanomechanical oscillator into its quantum ground state. *Nature*, *478*(7367), 89–92.
- Chaste, J., Eichler, A., Moser, J., Ceballos, G., Rurali, R., & Bachtold, A. (2012). A nanomechanical mass sensor with yoctogram resolution. *Nature nanotechnology*, *7*(5), 301–304.
- Clerk, A. A., & Marquardt, F. (2014). Basic theory of cavity optomechanics. *Cavity Optomechanics: Nano-and Micromechanical Resonators Interacting with Light*, 5–23.
- Ding, L., Baker, C., Senellart, P., Lemaitre, A., Ducci, S., Leo, G., & Favero, I. (2010). High frequency gas nano-optomechanical disk resonator. *Physical review letters*, *105*(26), 263903.
- Dobrindt, J. M., Wilson-Rae, I., & Kippenberg, T. J. (2008). Parametric normal-mode splitting in cavity optomechanics. *Physical Review Letters*, *101*(26), 263602.
- Dohn, S., Svendsen, W., Boisen, A., & Hansen, O. (2007). Mass and position determination of attached particles on cantilever based mass sensors. *Review of Scientific Instruments*, *78*(10), 103303.
- Dong, C., Fiore, V., Kuzyk, M. C., & Wang, H. (2012). Optomechanical dark mode. *Science*, *338*(6114), 1609–1613.
- Eichenfield, M., Chan, J., Camacho, R. M., Vahala, K. J., & Painter, O. (2009). Optomechanical crystals. *nature*, *462*(7269), 78–82.
- Fiore, V., Yang, Y., Kuzyk, M. C., Barbour, R., Tian, L., & Wang, H. (2011). Storing optical information as a mechanical excitation in a silica optomechanical resonator. *Physical review letters*, *107*(13), 133601.
- Forstner, S., Prams, S., Knittel, J., Van Ooijen, E., Swaim, J., Harris, G., ... Rubinsztein-Dunlop, H. (2012). Cavity optomechanical magnetometer. *Physical review letters*, *108*(12), 120801.
- Galeotti, F., Seršić Vollenbroek, I., Petruzzella, M., Pagliano, F., van Otten, F. W., Zobenica, Ž., ... Fiore, A. (2020). On-chip waveguide-coupled opto-electromechanical system for nanoscale displacement sensing. *APL Photonics*, *5*(2), 026103.

- Gardiner, C. W., & Collett, M. J. (1985). Input and output in damped quantum systems: Quantum stochastic differential equations and the master equation. *Physical Review A*, *31*(6), 3761.
- Gavartin, E., Verlot, P., & Kippenberg, T. J. (2012). A hybrid on-chip optomechanical transducer for ultrasensitive force measurements. *Nature nanotechnology*, *7*(8), 509–514.
- Gefen, T., Rotem, A., & Retzker, A. (2019). Overcoming resolution limits with quantum sensing. *Nature communications*, *10*(1), 4992.
- Giessibl, F. J. (2003). Advances in atomic force microscopy. *Reviews of modern physics*, *75*(3), 949.
- Griffiths, J. (2008). A brief history of mass spectrometry. *Anal. Chem*, *80*(15), 5678–5683.
- Guggenheim, J. A., Li, J., Allen, T. J., Colchester, R. J., Noimark, S., Ogunlade, O., ... others (2017). Ultrasensitive plano-concave optical microresonators for ultrasound sensing. *Nature Photonics*, *11*(11), 714–719.
- Guzmán Cervantes, F., Kumanchik, L., Pratt, J., & Taylor, J. M. (2014). High sensitivity optomechanical reference accelerometer over 10 khz. *Applied Physics Letters*, *104*(22), 221111.
- Hoff, U. B., Harris, G. I., Madsen, L. S., Kerdoncuff, H., Lassen, M., Nielsen, B. M., ... Andersen, U. L. (2013). Quantum-enhanced micromechanical displacement sensitivity. *Optics letters*, *38*(9), 1413–1415.
- Hu, Y.-W., Xiao, Y.-F., Liu, Y.-C., & Gong, Q. (2013). Optomechanical sensing with on-chip microcavities. *Frontiers of Physics*, *8*, 475–490.
- Kampel, N., Peterson, R., Fischer, R., Yu, P.-L., Cicak, K., Simmonds, R., ... Regal, C. (2017). Improving broadband displacement detection with quantum correlations. *Physical Review X*, *7*(2), 021008.
- Kim, P., Hauer, B., Clark, T., Fani Sani, F., Freeman, M., & Davis, J. (2017). Magnetic actuation and feedback cooling of a cavity optomechanical torque sensor. *Nature communications*, *8*(1), 1355.
- Krause, A. G., Winger, M., Blasius, T. D., Lin, Q., & Painter, O. (2012). A high-resolution microchip optomechanical accelerometer. *Nature Photonics*, *6*(11), 768–772.
- Law, C. (1995). Interaction between a moving mirror and radiation pressure: A hamiltonian formulation. *Physical Review A*, *51*(3), 2537.

- Lecocq, F., Clark, J. B., Simmonds, R. W., Aumentado, J., & Teufel, J. D. (2015). Quantum nondemolition measurement of a nonclassical state of a massive object. *Physical Review X*, 5(4), 041037.
- Li, J.-J., & Zhu, K.-D. (2013). All-optical mass sensing with coupled mechanical resonator systems. *Physics Reports*, 525(3), 223–254.
- Mason, D., Chen, J., Rossi, M., Tsaturyan, Y., & Schliesser, A. (2019). Continuous force and displacement measurement below the standard quantum limit. *Nature Physics*, 15(8), 745–749.
- Metcalf, M. (2014). Applications of cavity optomechanics. *Applied Physics Reviews*, 1(3), 031105.
- O’Connell, A. D., Hofheinz, M., Ansmann, M., Bialczak, R. C., Lenander, M., Lucero, E., ... others (2010). Quantum ground state and single-phonon control of a mechanical resonator. *Nature*, 464(7289), 697–703.
- Rabenimanana, T., Walter, V., Kacem, N., Le Moal, P., Bourbon, G., & Lardies, J. (2019). Mass sensor using mode localization in two weakly coupled mems cantilevers with different lengths: Design and experimental model validation. *Sensors and Actuators A: Physical*, 295, 643–652.
- Riedinger, R., Wallucks, A., Marinković, I., Löschnauer, C., Aspelmeyer, M., Hong, S., & Gröblacher, S. (2018). Remote quantum entanglement between two micromechanical oscillators. *Nature*, 556(7702), 473–477.
- Sansa, M., Defoort, M., Brenac, A., Hermouet, M., Banniard, L., Fafin, A., ... others (2020). Optomechanical mass spectrometry. *Nature Communications*, 11(1), 3781.
- Schliesser, A., Anetsberger, G., Rivière, R., Arcizet, O., & Kippenberg, T. J. (2008). High-sensitivity monitoring of micromechanical vibration using optical whispering gallery mode resonators. *New Journal of Physics*, 10(9), 095015.
- Stannigel, K., Komar, P., Habraken, S., Bennett, S., Lukin, M. D., Zoller, P., & Rabl, P. (2012). Optomechanical quantum information processing with photons and phonons. *Physical review letters*, 109(1), 013603.
- Tadigadapa, S., & Mateti, K. (2009). Piezoelectric mems sensors: state-of-the-art and perspectives. *Measurement Science and technology*, 20(9), 092001.
- Tallur, S., & Bhave, S. A. (2013). Rayleigh scattering boosted multi-ghz displacement sensitivity in whispering gallery opto-mechanical resonators. *Optics express*, 21(23),

27780–27788.

- Wollman, E. E., Lei, C., Weinstein, A., Suh, J., Kronwald, A., Marquardt, F., . . . Schwab, K. (2015). Quantum squeezing of motion in a mechanical resonator. *Science*, *349*(6251), 952–955.
- Xu, H., Jiang, L., Clerk, A., & Harris, J. (2019). Nonreciprocal control and cooling of phonon modes in an optomechanical system. *Nature*, *568*(7750), 65–69.
- Xu, H., Mason, D., Jiang, L., & Harris, J. (2016). Topological energy transfer in an optomechanical system with exceptional points. *Nature*, *537*(7618), 80–83.
- Yu, W., Jiang, W. C., Lin, Q., & Lu, T. (2016). Cavity optomechanical spring sensing of single molecules. *Nature communications*, *7*(1), 12311.
- Zhu, N., Zhang, X., Han, X., Zou, C.-L., Zhong, C., Wang, C.-H., . . . Tang, H. X. (2020). Waveguide cavity optomagnonics for microwave-to-optics conversion. *Optica*, *7*(10), 1291–1297.

## APPENDIX

Below is wolfram mathematical soft wares code I used to draw the graph.

syntax

clear all

close all

clc

Resonance frequency vesus effective mass

$M_{eff}=[1.5 \times 10^{-13}, 6.0 \times 10^{-8}];$

$k=[10]$

Plot [ $f_m = \frac{1}{6.28} \sqrt{\frac{10}{M_{eff}}}$ ,  $M_{eff}, 1.5 \times 10^{-13}, 6 \times 10^{-8}$ ]

*Zeropoint fluctuation*( $X_{ZPF}$ ) versus the optical (mechanical) resonance frequency ( $\omega_m$ )

$\omega_m=[10^{-6}, 100]$

$\hbar = [10^{-34}]$

$M_{eff} = [6 \times 10^{-8}]$

Plot [ $X_{ZPF} = \sqrt{\frac{\hbar}{2m_{eff}\omega_m}}$ ,  $\omega, 10^{-6}, 100]$